TOWARDS UNDERSTANDING ORIGIN OF UNIVERSE BY CONNECTING VEDIC CONCEPTS WITH MODERN BIG BANG THEORY: A TRIFOLD STUDY

A Thesis

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BY

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UNDER THE GUIDANCE OF

Late Dr. Shripad Bhat

SHREE BALMUKUNDA LOHIA CENTRE OF

SANSKRIT AND INDOLOGICAL STUDIES

Year - 2019

CERTIFICATE OF THE SUPERVISOR

It is certified that work entitled **'Towards Understanding Origin of Universe by Connecting Vedic Concepts with Modern Big Bang Theory: A Trifold Study'** is an original research work done by **Aditya Shankar Abhyankar** Under my supervision for the degree of Doctor of Philosophy in **Sanskrit** to be awarded by Tilak Maharashtra Vidyapeeth, Pune. To best of my knowledge this thesis

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Dedication

This thesis is dedicated to my mother (आई) .....

आई त्वया कृपेने। आदित्य लिहित आहे।
पुष्पे प्रबंध रूपी। तव चरणी अर्पिताहे॥

संशोधनाची सृष्टी। मज दाविलीस तू ते।
संशोधकाची वृत्ती। मज साक्षी तू स्वभावे॥

जिज्ञासु चक्षु दृष्टी। अभ्यास कर्म पुष्टि।
सत्यार्थ ज्ञान तुष्टि। शिकलो तुवा कराने॥

ममता त्वया मनीची। मज भक्ती मर्म ठावे।
समता तुवा मनीची। मज योग कर्म भावे॥

उकलून गूढ गोष्टी। विश्वात्म सृजन सृष्टी।
ज्ञानार्थ दीप लावी। तुझिया कृपे वराने॥

गति अल्प मम मतीची। चरणी तुझ्याच नमने।
श्रीराम वोळताहि। सुकर सर्व सुमने॥

In Memory of

This thesis is in the memory of Late Dr. Shripad Bhat (कै. श्रीपाद भट), my guide - advisor - mentor in this journey. Loosing him mid-way is unfortunate and the loss is irreversible. This thesis is in loving memory of my Gurū.

कै. श्रीपाद भट सरांच्या पवित्र स्मृतीस अभिवादन ...

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I am grateful and indebted to all those who have helped me in this endeavor either directly or indirectly.

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I am truly blessed to have the best possible family one could ask for. My father (विद्यावाचस्पति शङ्कर अभ्यङ्कर) is a living legend and my Sadgurū as well. My mother (Mrs Aparna S. Abhyankar (सौ. अपर्णा शङ्कर अभ्यङ्कर)) is the source of inspiration behind this work and I dedicate this thesis to her. It is only because of her sacrifices that I am what I am today. I am thankful to Dādā (CA Jitendra Abhyankar), Vahinī (Mrs. Madhura Abhyankar), my loving wife Āratī and the next generation of Abhyankars - my niece Arūndhatī, my nephew Arjuna and my son Advaita!

I am thankful to all my Gurūs and particularly the ones at Tilak Maharashtra Vidyapeetha. I will remain indebted to Prof. Mangala Mirasdar, Prof. Hema Dole, Prof. Vinaya Deo, Prof. Manjiri Bhalerao and also the non-teaching support staff at Sanskrit Department of Tilak Maharashtra Vidyapeetha.

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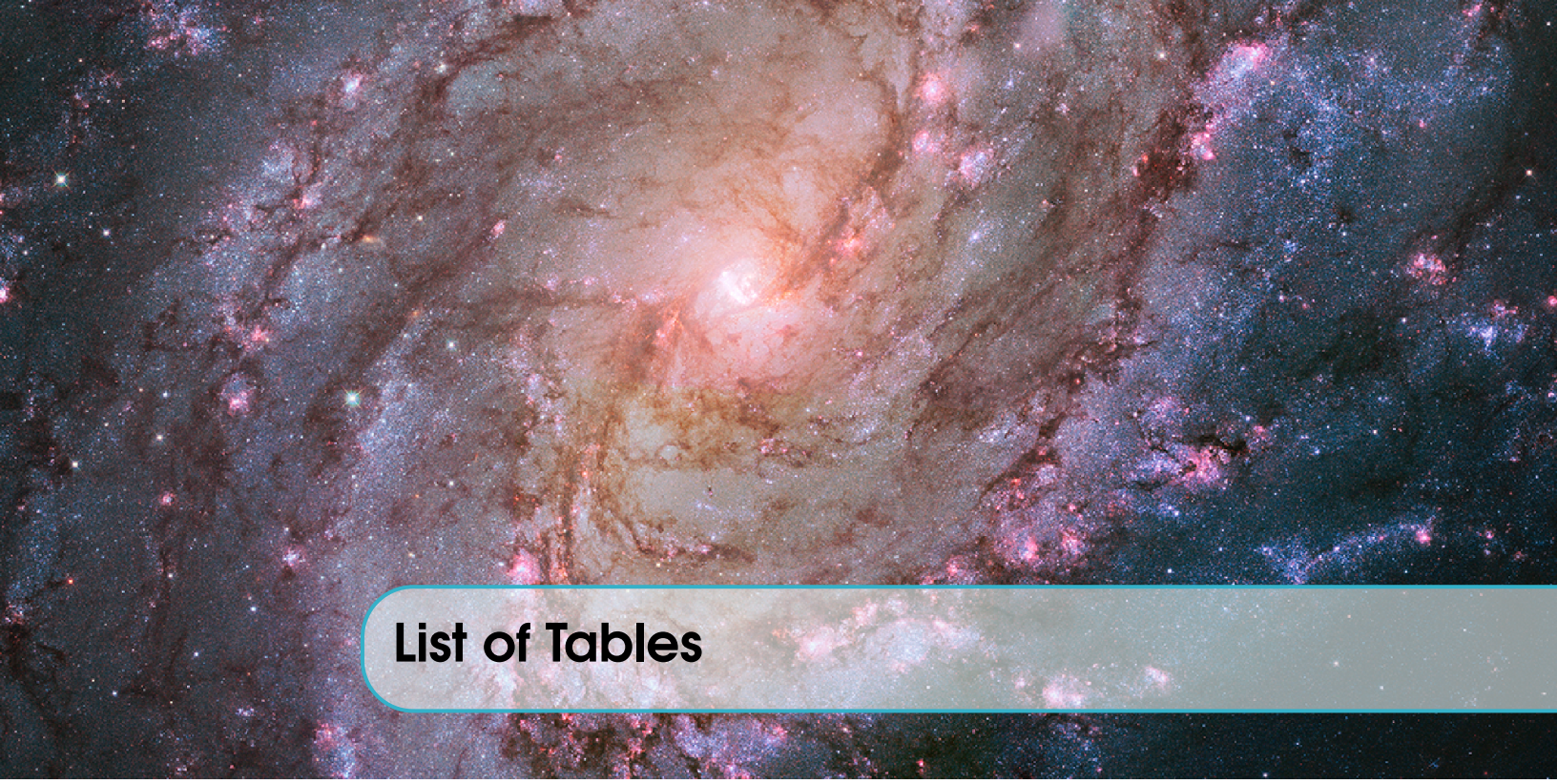
Finally I am thankful to Īśvara (ईश्वर) without wish of whom not even an atom moves, he made me his instrument in getting this work carried out. I hope this research will motivate other researchers to explore further and unlock more interesting mysteries to bring out diamonds from 'Bhāratīya Vāṅgamaya' (भारतीय वाङ्मय).

Contents



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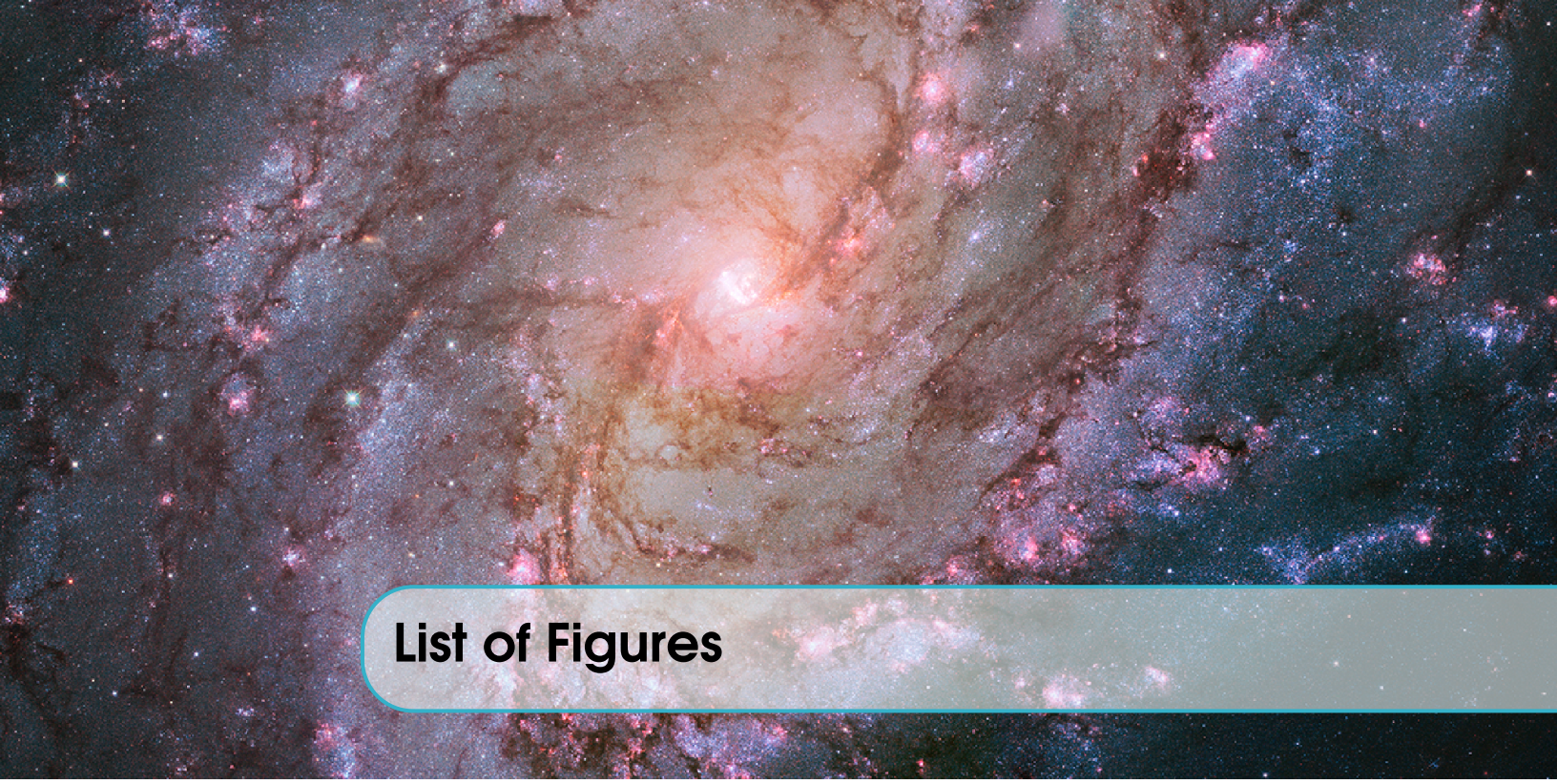

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1. Introduction

1.1 Introduction, Research Problem and Objectives

- Origin of the universe has been a long researched and yet an open ended problem.
- The problem not only brings in connectionism at single layer but makes the whole framework multilayered.
- Every single layer is sub-layered, thus making the very nature of the problem multidimensional.
- The systematic scientific discoveries in the modern science and technology strangely and uniquely correlates with Vedic (Vaidika = वैदिक) optics and work-flows.
- This creates a need and desire to dig deep to bring out such highly correlated pieces and study the patterns for connecting the dots.

Analytics 1.1.1 — The-Big-Bang-Theory.

- The Big Bang theory is an effort to explain what happened at the very beginning of our universe. Discoveries in astronomy and physics have shown beyond a reasonable doubt that our universe did in fact have a beginning. Prior to that moment there was nothing; during and after that moment there was something: 'our universe'. The big bang theory is an effort to explain what happened during and after that moment ⟦1⟧.
- According to the standard theory, our universe sprang into existence as "singularity" around 13.7 billion years ago. What is a "singularity" and where does it come from? Well, to be honest, we don't know for sure. Singularities are zones which defy our current understanding of physics. They are thought to exist at the core of "black holes". Black holes are areas of intense gravitational pressure. The pressure is thought to be so intense that finite matter is actually squished into infinite density (a mathematical concept which truly boggles the mind). These zones of infinite density are called "singularities". Our universe is thought to have begun as an infinitesimally small, infinitely hot, infinitely dense, something - a singularity. Where did it come from? Why did it appear? ⟦2⟧.
- After its initial appearance, it apparently inflated (the "Big Bang"), expanded and cooled, going from very, very small and very, very hot, to the size and temperature of our current

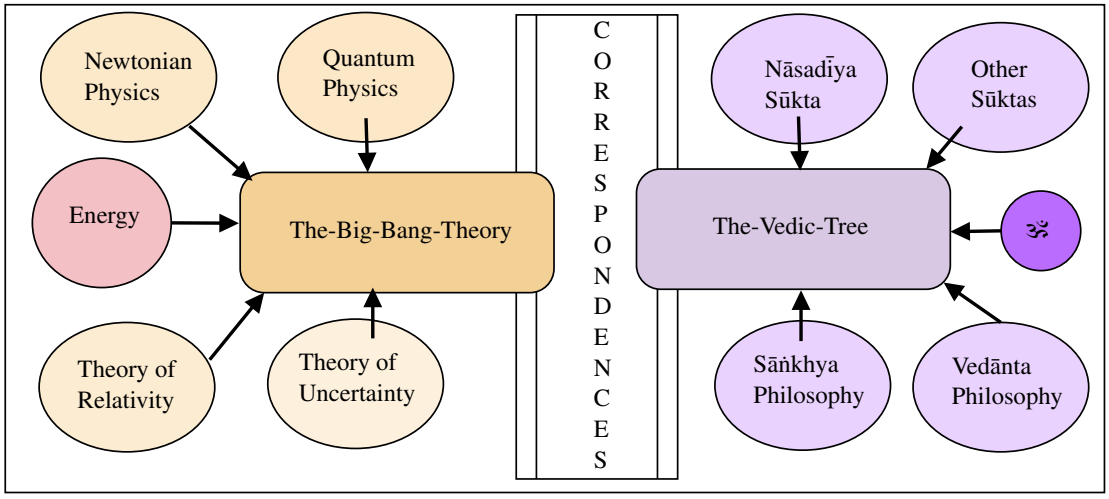
universe. It continues to expand and cool to this day and we are inside of it: incredible creatures living on a unique planet, circling a beautiful star clustered together with several hundred billion other stars in a galaxy soaring through the cosmos, all of which is inside of an expanding universe that began as an infinitesimal singularity which appeared out of nowhere for reasons unknown. This is the Big Bang theory in Nutshell [3].

Analytics 1.1.2 — Vedic Conceptualization.

- Even since immortal times of Vedas, Origin of Universe has been one curious and open ended question. There are instances of subtle mention about the origin of universe, however these are encrypted forms.
- We develop Vedic connects by keeping 'Nāsadīya Sūkta' (नासदीय सूक्त) from Ṛg Veda (ऋग्वेद) at the heart of the study.
- With this at the heart, we propose to build the Vedic equivalent tree for bringing correspondences through correlation with Big Bang Theory.
- This Vedic tree would consist of other hymns and verse from Ṛg Veda (ऋग्वेद) as well as other Vedas, along with Saṅkhya (साङ्ख्य) and Advaita Vedānta (अद्वैत वेदान्त) implementations.

1.1.1 Schema

The research schema is as shown in diagram below:



- The schema of this work is being put down.
- We intend to build the Modern-Science-Tree (MST).
- This MST includes Energy Analysis, Newtonian Physics, Quantum Physics, Theory of Relativity by Einstein, Theory of Uncertainty and Schrödinger's theories.
- On the other side we will build the The-Vedic-Tree (VT).
- This VT includes Nāsadīya Sūkta, Other Sūktas, Sāṅkhya Philosophy, Vedānta Philosophy with the foundation of that of Praṇava (प्रणव) or ॐ.
- We intend to bring out the correspondences between MST and VT.
- Our conclusions will be based on the detailed analysis carried out.

1.2 Significance of the Proposed Research Study

- Beginning and development of Universe has been a long researched and yet an open ended issue.
- While few cryptic hymns from Vedic literature are devoted for describing the beginning, it is needed to decrypt these scripts scientifically to make sense.
- It is equally desired to connect the findings from these Vedic hymns with the modern effort of Big Bang Theory.
- Such efforts has not been done in the past to the best of our knowledge and while the newer finding from Big Bang are becoming prevalent, such effort shall only help enhance the understanding of human race about our universe.
- This study shall be done without any preconceived notions and only through systematic and scientific methods.

1.3 Research Problem

- Although the two domains have been studied discretely, there is a need to do it jointly, systematically and analytically.
- While few cryptic hymns from Vedic literature are devoted for describing the beginning, it is needed to decrypt these scripts scientifically to make sense. One Example is shown below:

न मृत्युरासीदमृतं न तर्हि न रात्र्या अह्न आसीत्प्रकेतः ।
आनीदवातं स्वधया तदेकं तस्माद्धान्यन्न परः किं चनास ॥ 2 ॥
तम आसीत्तमसा गूळ्हमग्रेऽप्रकेतं सलिलं सर्वमा इदम् ।
तुच्छ्येनाभ्वपिहितं यदासीत्तपसस्तन्महिनाजायतैकम् ॥ 3 ॥

- Most of the research that has taken place is uni-modal
- There is scope to go beyond and study the linkages, thus creating a multi-modal system.

1.4 Aims and Objectives of the Proposed Research Study

Analytics 1.4.1 — Aims and Objectives.

- Building Vedic Big Bang Tree with 'Nāsadīya Sūkta' as reference point
- Creating an adaptive multimodal framework for studies at Physical, Conceptual and Temporal levels.
- Testing the hypothesis using modern parametric and non-parametric methods.
- To Benchmark the findings.

1.5 Research Questions

- How the Universe was formed?
- What are the corresponding evidences from Vedic Literature?
- What efforts have gone into connecting the two theories?
- Are the results sequestered?

- What level of accuracy is desired and how much is the gap between desired and available levels?
- Will enhancement in these results help elevate human understanding of the problem at heart?

1.6 Hypothesis the Proposed Research Study

Analytics 1.6.1 — Hypothesis.

- We hypothecate to carry out the systematic research at three different levels as follows:
  1. Physical Level
  2. Conceptual Level
  3. Temporal Level
- Physical Level:
  1. Hypothecation is based on similarities observed, e.g. between 'Skambha devatā' from Atharva-Veda and Hydrogen from modern chemistry.
  2. The hypothesis is: There are a little too many such coincidences. From incidentals to coincidental, there must be a crisp logic beneath!
- Conceptual Level:
  1. Theoretical Advancements
  2. Understanding Evolutionary Patterns
  3. Scientific meaning of 'Sat' and 'Asat'.
  4. Analyzing concepts like पञ्चीकरण etc
  5. Figuring out noticed similarities like 3 leads of modern Transistor and Brahmā, Viṣṇu and Maheśa.
- Temporal Level:
  1. Similarities noticed in modern physics and 'Purāṇas'.
  2. Time keeping with reference to 'Purāṇas' and 'Vedas'.
  3. Time Evolution of the creation of the Universe.

1.7 Research Design

- Most of the research that has taken place is uni-modal.
- There is scope to go beyond and study the linkages, thus creating a multi-modal system.
- Hence we propose to carry out the systematic research at three different levels.
- The ground truth shall be established by annotation through experts.
- Statistical methods shall be used to do the analysis.

1.8 Research Methodology and Techniques

- Data Collection from both the domains
- Annotation of data from domain experts
- Building consensus using inter-rater statistics
- Creating the ground truth
- Training the supervised BP NN for non-linear, non-stationary and non-parametric solution
- Creating fuzzy c-means graphs
- Drawing conclusions based on these
- Cross-Validation

1.9 Research Approach

- Multi-modal approach for enhancing understanding of origin of universe
- Building connectionism amongst Vedic hymns and Big Bang Theory.
- Enhancing, Enriching and Embellishing human understanding of Universe.
- Bringing out scientific importance of Vedic hymns.
- Betterment in correlation and correspondences between modern and vedic schools of thoughts.

1.10 Scope and Limitations

This sections describes the scope and limitations of this work.

- This study uses energy analysis for bringing out correspondences between Modern Science Tree and Vedic Tree.
- There are several other means of bringing out the correlation, however this study focuses on the energy analytics that emerges out of basic of signal processing.
- This study touches allied branches of science and spirituality, but the focus remains on the energy analysis to build the correspondence.
- The multiple other ways of building the correlation exist (e.g. numerical system, functional system), however in this work these pathways are not explored to the fullest and they fall beyond the purview of this work.
- The characterization of the systems is also restricted to signal processing realms.
- The authors certainly respect and value these other pathways, however to maintain the focus and crispness in the analysis, it was decided consciously to concentrate on the signal processing aspects of the study.

1.11 Chapter-wise Plan

This thesis is divided into five chapters. The names, description and short summary of all these chapters is as follows

1. **Introduction:**
   This chapter introduces the research problem of this work to the readers. This chapter also details down the research schema, methodology adapted, scope and limitations of the work. This chapter also describes the hypothesis emerging out of the basic research problems beneath.
2. **Survey of the Research:**
   Any research is incremental in nature and is based on the work done by others. This chapter systematically brings out the literature survey and converts into literature review. The potential areas of research are also brought out to justify the hypothesis presented in the first chapter.
3. **Methodology:**
   This is an important chapter of the thesis and presents the main part of the research carried out. This chapter provides the interpretation of the decoding on the verses from Nāsadīya Sūkta (नासदीय सूक्त). This chapter provides comprehensive analysis of every word and presents a novel style of decoding the verses. The correspondences are drawn between the decoded verses and the modern postulates of signal processing. The chapter also delves into few other important Sūktas to make the presentation more comprehensive and robust.
4. **Results, Analysis and Discussions:**
   This chapter takes an important thread from the previous chapter further. From verse 4 and 5 of Nāsadīya Sūkta (नासदीय सूक्त) the doctrine of Satkāryavāda (सत्कार्यवाद) gets established.

The two important schools of thoughts out of the Ṣaṭdasśanas (षट्दर्शने) or the six schools of Indian Philosophy which prescribe the concept of Satkāryavāda (सत्कार्यवाद) to it's potential are:

(a) Sāṅkhya (साङ्ख्य)
(b) Vedānta (वेदान्त) or Advaita (अद्वैत)

This chapter presents the signal processing experimentation carried out on these two schools of thoughts. The mathematical results are supported with simulations. The results are discussed in detail to provide insights into the subject matter.

5. **Concluding Remarks:**
This chapter presents the conclusions drawn out of the experimentation carried out. The chapter presents the contributions which emerge out of the analysis presented in this thesis and the advancements proposed. The chapter also presents the future scope for furthering this research work.

2. Survey of the Research

2.1 Survey Overview

Any research is incremental and literature survey followed by review helps in understanding the state-of-the-art of that specific piece of research. The review of research also enables in understanding how other researchers have approached that research problem. This chapter provides detailed literature review with identified research areas. The review of the research is done in three areas:

- Origin of the universe as per the modern science
- Viśvotpattī (विश्वोत्पत्ती) that is the creation of the universe as per Vaidika (वैदिक) tradition.
- The correspondences drawn between the above two approaches.

To review the literature, we refer to the research schema as put down in chapter 1.

2.2 Origin of the universe as per modern science

The literature is reviewed to build the Modern-Science-Tree (MST). At the heart of this MST remains The-Big-Bang-Theory and allied theories like Theory of Relativity by Albert Einstein, Classical Physics by Newton and Theory of Uncertainty by Heisenberg revolve around it. The following four manuscripts had lasting impact on the author to think more of the subject matter:

- Tao of Physics by Fritjof Capra [4]
- Dancing Wu Li Masters [5]
- Lost Discoveries by Dick Teresi [6]
- Cosmos by Carl Sagan [2]

This triggered curiosity was the genesis for the literature survey presented in Table (2.1) below:

These eleven readings formulate the core of the Modern-Science-Tree (MST). There are other pieces of literature which were also referred and it includes [18], [19], [1], [3], [20], [21], [22], [23].

From the literature read and comprehended following important pointers were derived:

Sr No	Name of the manuscript and Author	Name of the Publisher	Year of Publication	Short Review Summary
1 [7]	The universe and Dr. Einstein by L Barnett	Harper and Row, NY	1948	This research discusses the relativity and how Einstein looked at the universe and it's emergence. The manuscripts also connects science with spirituality towards the end from the prespective of Einstein
2 [8]	Atomic Theory and description of nature by Neil Bohr	Cambridge University Press, England	1934	This manuscript discusses about the very core of an atom and how microscopic phenomenon are observed at macro level. This connects the atomic constructs with the laws of nature that build this universe.
3 [9]	On Physical Reality by Albert Einstein	Franklin Institute Journal	1936	This journal article by Einstein himself discusses and presents the physicist's perspective towards the making of the universe.
4 [10]	Mathematical formulation of Quantum Theory of Electromagnetic Interactions	Schwmeger - Selected papers, New York, Dover	1958	This Quantum formulation of modern physics enables one to explore probabilistic domain to characterize the very notion of creation and understanding of the universe.
5 [11]	Singularities in the Geometry of Space-time by Stephen Hawking	Adams Prize, Cambridge University	1966	This very famous article by Hawking presents the thematic understanding of singularities. The space-time unison and
6 [12]	Physics and Philosophy by Werner Heisenberg	Harper and Row, NY	1968	This manuscript by the scientist who invented the uncertainty principle is a wonderful treatise on the unison of science and spirituality
7 [13]	Philosophiae Naturalu, Principle Mathematica trans Andrew Motte, reprinted in Sir Isaac Newton's Mathematical Principles of Natural Philosophy and His System of the World (revised trans Flonan Cajon)	Berkley University, California Press	1946	This reprint of the original work by Newton is an important piece in underatanding the mathematical construct of the Newtonian Physics which dominated the scene before emergence of the Quantum Physics. Newton not only presents the laws of nature in his unique style, but also explores the connect with the realms of philosophy and gives his system of world. This is a profound thesis of analysing the giving the cause and existence of the very universe. This is certainly an important milestone in presenting the foundation of all future attempts.
8 [14]	The Philosophy of Physics by Max Plank	Norton, New York	1936	Max Planks is considered as father of Quantum Mechanics. His work brings out paradigm shift in the theory proposed by Newton
9 [15]	Image of Matter by E Schrodinger	Clarkson Potter, New York	1961	Schrodinger's Equation is considered to be an enhanced version of Theory of Relativity by Einstein. This work presents his mysterious equation and the annotations
10 [16]	Black Hole: The End of Universe by J Taylor	Random House, New York,	1973	In the early 80s the topic of Black Holes was discussed widely across the globe. This manuscript presents detailed description of understanding Black Holes in a very scientific way
11 [17]	The Mathematical Fooundation of Quantum Mechanics by Von Neumann	Princeton University Press, Princeton	1955	Von Neumann is considered as father of modern computing. His writings on the Quantum Mechanics and very precise. He also presents the big picture of connecting this theory with the universal laws and understanding them.

Table 2.1: Literature Review of Modern Science on Universe

- There are various theories about 'Creation of Universe', and yet it still is an open ended problem and has good research potential.
- The scientists and technocrats have used modern science and technology to decode the mystery behind genesis of Universe effectively.
- Almost unanimously, these scientists have eventually converged with the philosophical and spiritual realms.
- This leads to a strong case with good potential to explore science and spirituality together.
- The notional understanding of universe by these scientists is almost similar although outwardly the manifestation may appear different.

2.3 Origin of the universe as per Vaidika Literature

This builds a strong case to explore the Vedic-Tree literature further. The heart and core of this is Nāsadīya Sūkta (नासदीय सूक्त).

Nāsadīya Sūkta (नासदीय सूक्त) has seven hymns. For every hymn we take into consideration commentaries provided by 15 scholars across the globe. These scholars are from vivid religious, geographical, cultural backgrounds and everyone adds value to our understanding. It should be mentioned however, the interpretations given by Swamī Vivekānanda, Sāyaṇācārya and others who follow them are closest to our heart. The interpretations of these scholars are provided as a ready reference before giving our decoding of the hymn. Following Table (2.2) provides the list of these 15 scholars. The interpretations presented by Dr. Bheemarao S Kulkarni [24] and Shri. R G Kolangade [25] were also studied along with the work done by Āhitāgni Rājawāḍe (आहिताग्नि राजवाडे) [26].

Various interpretations of Nāsadīya Sūkta are studied in depth and the new interpretations are presented in this thesis. Along with this, other literature including [42], [43], [44], [45], [46].

The study of these various interpretations gives following leads:

- Most of the interpreters are either philosophers or philanthropists from various regions of India
- There are very few interpreters who bring confluence of Vedika scriptures with modern science
- Although the interpretations provide very strong hint of all the hymns being very scientific, there is a need to bring the same out systematically.
- There are certain studies where very tall claims of one can know everything is Nāsadīya Sūkta is understood are made, however there are no concrete doctrine to support the same.

2.4 Literature on combined approaches

The third category of survey of literature presents the works where Nāsadīya Sūkta and Vaidika literature is studied along with modern science to understand origin of universe. In this study only scientific literature is considered, and multiple blogs on internet about the Nāsadīya Sūkta and creation of universe are not considered.

The important manuscripts studied along with the research potential in it is presented in Table (2.3) below. This tabular presentation has one exemplar from Vaidika and one from modern literature.

Few more manuscripts comprehended under this category also include: [47], [48], [49], [50], [51], [52], [53], [54], [55]. Amongst these, the work by Dr. Apte [56] is worth special mention.

List of Comparative Interpretations of scholars on Hymns of Nasadiya Sukta	
Name of the interpreter	Title, Publisher and Year
Vivekananda [27]	The Complete Works of Swami Vivekananda, Volume 6 by RamKrishna Mission, 1892.
Max Mueller [28]	History of Ancient Sanskrit Literature, Williams and Norgate, 1860
A. A. Macdonell [29]	A History Of Sanskrit Literature, Motilal Banarasidas, New Delhi, 1965
Raimundo Panikkar [30]	The Vedic Experience - Mantra Manjari, Motilal Banarasidas, 1977
Griffith [31]	The Hymns of the Rigveda - English Translation of the Rigved, Motilal Banarasidas, 1976
Jamison & Brereton [32]	Edifying Puzzlement: Rgveda 10.129 and the Uses of Enigma, Journal of the American Oriental Society, 1999
Geldner [33]	Der Rig-Veda, Rotshchild publishers, 1951
A. L. Basham [34]	The Wonder That Was India, UK, 1954
Wilson, HH: [35]	Rig-Veda Sanhita: A Collection of Ancient Hindu Hymns, UK, 1850
Wendy Doniger O'Flaherty [36]	The Rig Veda: An Anthology, Penguin, New York, 1981
Pandit RamGovind Trivedi [37]	Hindi RigVeda, Indian Press Limited, Prayag, 1954
Siddheshwar Shastri Chitrav [38]	RigVedache Marathi Bhashantar, Bharatiya CharitraKosha Mandal, Pune, 1996
Krishnananda [39]	Nasadiya Sukta, 2004
Hari Damodar Velankar [40]	Ṛksūktavaijayantī, Vaidika Samshodhana Mandala, Pune, 2009
Sāyaṇācārya [41]	Ṛgveda-Saṁhitā with the commentary of Sāyaṇācārya, Vaidika Samshodhana Mandala, Pune, 1946

Table 2.2: Contemporary Interpretations of Hymns of Nāsadīya Sūkta

Sr No	Title and Author	Limitations	Research Potential
1	Nasadiya Sukta Bhashya - Ahitagni Rajwade	The interpretations are skewed. In particular, hymn 4 is interpreted in a manner that changes the course of further study.	There is need to interprete hymn 4 and for that matter all the hymns in more scientific manner. The signal processing perspective is missing which can br brought out.
2	The Cosmic Connection: An Extraterrestrial Perspective - Carl Sagan	The study provides deep interpretations on the cosmology. The balance between modern and conventional literature is missing	There is good potential to explore cosmology from the very fundamental perspective as provided in Vaidika literature. A good balance between vaidika and modern literature may lead to novel pathways.

Table 2.3: Critical Literature Pointers - Research Potential

2.5 Summary

The summary of the literature surveyed and reviewed is as follows:

- There is need to combine modern science with Vaidika science in balanced way to understand the creation of universe better.
- There are very few attempts made on these lines and most of these attempts are skewed.
- There is very little done using energy analysis using the signal processing perspective.
- Energy analysis from signal processing perspective can be identified as potential area of research on the said topic.
- The comprehension of cosmogenesis may get evolved by combining the best of modern and Vaidika principles.

3. Methodology: 'Nāsadīya Sūkta'

3.1 Introduction: Towards Deciphering 'Nāsadīya Sūkta'

Nāsadīya Sūkta (नासदीय सूक्त) appears as 129^{th} in the 10^{th} Maṇḍala (मण्डल) of ṚgVeda (ऋग्वेद). There are total seven Ṛcās (ऋचा) composed in triṣṭupa (त्रिष्टुप) meter or chanda (छन्द). This section provides our interpretation of these towards decoding the origin of the universe.

This Sūkta (सूक्त) begins with 'Nāsadāsīnnosadāsīt' (नासदासीन्नोसदासीत्) and hence the name. The Ṛṣi (ऋषि) of this sūkta (सूक्त) is prajāpati (प्रजापति) named parameṣṭhi (परमेष्ठि) and devatā (देवता) is paramātmā (परमात्मा).

R From the philosophical and spiritual perspective, this sūkta (सूक्त) is so enriching, that author genuinely thinks that it should have become an Upaniṣada (उपनिषद).

The language used in Nāsadīya Sūkta (नासदीय सूक्त) appears to be confusing for many scholars. The very subject of this sūkta (सूक्त) is to throw light on the very kernel of what is beyond the very realms of the physical world. The effort is to explain what is inexplicable! That is why, although the description sounds or looks and appears confusing, it is not. In fact, if understood appropriately, it leads one to enlightenment. The most crisp dictum of Nāsadīya Sūkta (नासदीय सूक्त) is - the physical world around us is of the form of Dvandva (द्वंद्व) and the creator is beyond the physical world and hence is Nirdvandva (निर्द्वंद्व). At metaphysical level this journey is from Dvaita (द्वैत) to Advaita (अद्वैत).

The Nāsadīya Sūkta (नासदीय सूक्त) in devanāgarī (देवनागरी) with Svara (स्वर) marked is as follows:

.. Nāsadīya Sūkta ..

.. नासदीय सूक्त ..

1. नासदासीन्नो सदासीत्तदानीं नासीद्रजो नो व्योमा परो यत् ।
   किमावरीवः कुह कस्य शर्मन्नम्भः किमासीद्गहनं गभीरम् ॥ १०.१२९.०१
2. न मृत्युरासीदमृतं न तर्हि न रात्र्या अह्न आसीत्प्रकेतः ।
   आनीदवातं स्वधया तदेकं तस्माद्धान्यन्न परः किं चनास ॥ १०.१२९.०२
3. तम आसीत्तमसा गूळ्हमग्रेऽप्रकेतं सलिलं सर्वमा इदम् ।
   तुच्छ्येनाभ्वपिहितं यदासीत्तपसस्तन्महिनाजायतैकम् ॥ १०.१२९.०३
4. कामस्तदग्रे समवर्तताधि मनसो रेतः प्रथमं तदासीत् ।
   सतो बन्धुमसति निरविन्दन्हृदि प्रतीष्या कवयो मनीषा ॥ १०.१२९.०४
5. तिरश्चीनो विततो रश्मिरेषामधः स्विदासीऽदुपरि स्विदासीऽत् ।
   रेतोधा आसन्महिमान आसन्त्स्वधा अवस्तात्प्रयतिः परस्तात् ॥ १०.१२९.०५
6. को अद्धा वेद क इह प्र वोचत्कुत आजाता कुत इयं विसृष्टिः ।
   अर्वाग्देवा अस्य विसर्जनेनाथा को वेद यत आबभूव ॥ १०.१२९.०६
7. इयं विसृष्टिर्यत आबभूव यदि वा दधे यदि वा न ।
   यो अस्याध्यक्षः परमे व्योमन्सो अङ्ग वेद यदि वा न वेद ॥ १०.१२९.०७

The devatā (देवता) of Nāsadīya Sūkta (नासदीय सूक्त) is paramātmā (परमात्मा). Paramātmā essentially means Paramatattva (परमतत्त्व) or the ultimate that is to be understood and comprehended. Thus, it is obvious that this sūkta (सूक्त) discusses the subject matter which is of the highest order and hence, the treatment and the way in which it is explained is also of the highest quality and extremely matured. It is then imperative that the seekers should elevate their maturity to understand the same.

3.2 Schema for comprehending Vedic Ṛcā

Ṛgveda (ऋग्वेद) is arguably the oldest doctrine in text form. It has been researched vividly by western scholars, Indian scholar and others too. The language of Ṛgveda (ऋग्वेद) is little cryptic and therefore in this work Ṛgveda (ऋग्वेद) is studied and looked upon as a **code-book**.

It is then important to come up with a systematic approach to decipher it. This and next few sections bring out the schema which is used for appropriate decoding. Before deciphering the hymns, first efforts put together by researchers in the same direction are discussed now.

The credit of taking the Vedic scriptures and making the western world familiarize with it goes to Colebrooke and he has produced extensive account of the same in [55]. Amongst earlier scholars from Germany who understood the importance of the study of Vedas, and Ṛgveda (ऋग्वेद) in particular was Adolf kaegi [57]. The work done by Max Müller is one of the most referred and sought after work [28] along with which under the guidance of his teacher Burnhof, he also went on to compile Ṛgveda Saṁhitā (ऋग्वेद संहिता) with the commentary of Sāyaṇācārya (सायणाचार्य). In the praise of the Indian culture and the advancements which were achieved at several levels, Arthur A Macdonell wrote [29]:

> **Quotation 3.2.1 — Macdonell.** "When the Greeks, towards the end of the fourth century B.C., invaded the North-West, the Indians had already fully worked out a national culture of their own,

unaffected by foreign influence. And in spite of successive waves of invasion and conquest by Persians, Greeks, Seythians, Muhammadans, the national development of the life and literature of the Indo-Aryan race remain practically unchecked and unmodified from without down to the era of British occupation. No other branch of the Indo-European stock as experienced an isolated evolution like this.

A few examples will serve to illustrate this remarkable continuity in Indian civilization. Sanskrit is still spoken as the tongue of the learned by thousands of Brahmans, as it was centuries before our era. Nor has it ceased to be used for literary purpose, for many books and journals written in the ancient language are still produced. The copying of Sanskrit manuscripts is still continued in hundreds of libraries in India, uninterrupted even by the introduction of printing during the present century. The Vedas are still learnt by heart as they were long before the invasion of Alexander, and could even now be restored from the lips of religious teachers if every manuscript or printed copy of them were destroyed."

The Vedas are the primitive knowledge source per Hindū tradition. The word Veda has its origins from the root 'Vid' (विद्) which essentially means knowledge or wisdom. This knowledge has been manifested through the wisdom, internal vision and intuition of the seers, which makes Vedas to be sacred knowledge bank. Conventionally Vedas have been considered to be Apauruṣeya (अपौरुषेय), that is existed from eternity with non-human and divine origin. The elevated minds of the Seers or Ṛṣis (ऋषी) could grasp it directly from the Supreme Being. As a result of the austerities performed, these seers could literally see the Verses or Ṛcās (ऋचा). The word Ṛcā (ऋचा) is derived from the root Ṛk (ऋक्) which clearly states that Ṛgveda (ऋग्वेद) is a codebook with sacred hymns. It is then important to understand the code to be able to decode the Ṛcās (ऋचा) in Ṛgveda (ऋग्वेद).

Mr N D Sonde [58] has provided excellent quotations and insight into the work of Whitehead [59]. It is worth revisiting and pondering over the following quotation of Whitehead provided by Sonde.

Quotation 3.2.2 — Sonde/Whitehead. "Veda is eternal, imperceivable, all pervading, all-comprehensive, exceedingly subtle, un-decaying, without beginning and without end entirety of wisdom though receptivity, reflection and constant meditation. It is what Dr. Whitehead designates 'as something which is real and yet waiting to be realized; something which is remote possibility and yet the greatest of present facts; something that gives meaning to all that passes; and yet alludes apprehension; something whose possession is the final good and yet beyond all reach; something which is ultimate ideal and the hopeless quest'. Veda is Wisdom of the unknown and not the Knowledge of the known"

The universality of Vedas is nicely captured by T N Dharmadhikari [60]. In his own words:

Quotation 3.2.3 — Dharmadhikari. "The word Veda therefore mens consideration, discussion and experience also. The cognate roots of the term Veda are also found even in European languages. Thus - Oida in Greek, Videre in Latin, Wissen in German and wit in English. If the word Veda is related to vedeo or wisdom or vision, it would mean DARŚANA. Etymologically also, the Vedas, therefore are seen internally by the Seers. The word Ṛgveda (ऋग्वेद) is also sometimes derived from the root dṛś (दृश्) to see."

Lokamānya Ṭilaka (लोकमान्य टिळक) provides astronomical analysis and refers to Śatapaṭha Brāhmaṇa (शतपथ ब्राह्मण) and Vedāṅga Jyotiṣa (वेदाङ्ग ज्योतिष), thus providing chronology in Vedas. He fixes period of Ṛgveda (ऋग्वेद) to be 4000 B.C. The very ancient nature of Vedas, primordial language and cryptic manifestation makes interpretation of Vedas extremely challenging.

It is of utmost importance though to understand the meaning of the the Ṛcās (ऋचा) in Ṛgveda (ऋग्वेद). Although the Ṛcās (ऋचा) are the Mantras (मंत्र) and when uttered using Swara (स्वर), namely udātta (उदात्त), anudātta (अनुदात्त) and swarita (स्वरित), it produces the effect of supreme order called Swara-Siddhī (स्वर-सिद्धी). Though this is true, Yāskācārya (यास्काचार्य) in his Nirukta (निरुक्त) emphatically proclaims, if the verses are only memorized without understanding the meaning behind, such person is a tree, a stump, a mere carrier of the load! But the difficult part is Vedic Saṃskṛta (संस्कृत) is different than its present day form. Hence, we take help of Nirukta (निरुक्त) of Yāskācārya (यास्काचार्य) to decipher it.

3.2.1 Nighaṇṭu / Nirukta of Yāskācārya

Nighaṇṭu (निघण्टु) = Vedic dictionary and Nirukta (निरुक्त) = commentary on Nighaṇṭu are master-pieces of Yāskācārya (यास्काचार्य) which dates back to 800 B.C. It is indeed one of the oldest and richest Indian texts which provides insight on how to understand Vedic hymns and has scientific discussion on Etymology, Semantics and Philology. Yāska (यास्क) provides commentary on some 600 hymns in Nirukta (निरुक्त). This is commentary on his own Nighaṇṭu (निघण्टु). Nighaṇṭu is divided into three sections [61]:

1. Nighaṇṭu Kāṇḍa (निघण्टु काण्ड) - provides synonyms
2. Naigama Kāṇḍa (नैगम काण्ड) - provides homonyms
3. Daivata Kāṇḍa (दैवत काण्ड) - section dedicated to deities

According to Yāska (यास्क), in (1-19) of Nirukta (निरुक्त), the sacrifice is flower and it fruitions into knowledge of deities! (अर्थं वाचः पुष्पफलमाह । याज्ञदैवते पुष्पफले । देवताध्यात्मे वा॥ निरुक्त १.१९). We therefore, focus more on the Daivata Kāṇḍa (दैवत काण्ड) which provides detailed account of vivid deities.

Yāska (यास्क) is very clear that the Mantras (मंत्र) are not recited for nothing and for unseen, but for the meaning! This is an example of Ṛcās 4.58.3 (ऋचा ४.५८.३).

- चत्वारि शृंगा त्रयो अस्य पादा, द्वे शीर्षे सप्त हस्तासो अस्य ।
  त्रिधा बद्धो, वृषभो रोरवीति महो देवो मर्त्यँआविवेश ॥ ऋग्वेद ४.५८.३

This hymn is of Agni (अग्नि). If the interpretation is not done appropriately it leads to confusion. The four horns are four Vedas, and the three legs are three worlds namely Bhūḥ (भूः), Bhuvaḥ (भुवः) and Swaḥ (स्वः). All the deities reside and operate in these three worlds! It is well established that they all originate from the one and only 'Sat' (सत्) and are manifestation of that alone. This is expressed as Ekam Sat Viprā bahudhā vadanti in Ṛgveda - 1.164.46 (एकम् सत् विप्रा बहुधा वदन्ति। ऋग्वेद - १.१६४.६४).

As Bloomfield [62] writes that Vedic deities to be representatives of nature objects, it becomes more obvious that Western scholars have difficulty understanding the true nature of the Vedic deities. They are also struggling to understand if this is Polytheistic or Monotheistic as is expressed by Max Müller [28]. These two quotations are reproduced as a ready reference below:

> **Quotation 3.2.4 — Bloomfield.** "For my part I always come to this theme in the spirit of scientific elation. You know from preceding statements what I meant by transparent gods. They are the gods who are at one and the same time nature object and person. In other words, they are mythic formations whose personification is arrested; by the continued action and the vivid memory of the very qualities which lead to personification. Figuratively speaking, just when the chemical is about to precipitate or to crystallize into something unrecognizable, and fare removed from its elements, it is shaken and dissolved anew. We are spared the labor of a qualitative and quantitative analysis."

Quotation 3.2.5 — Müller. "I could not even answer the question, if you were to ask whether the religion of the Veda was polytheistic, or monotheistic. Monotheistic, in the usual sense of that word, it is decidedly not though there are hymns that assert the unity of the Divine as fearlessly as any passage of the Old Testament, or the New Testament, or the Koran. Thus one poet says - That which is one, sages name it in various ways- they call it Agni, Yama, Matarisvan. Another poet says - the wise poets represent by their words Him who is one with beautiful wings, in many ways. And again we hear of a being called Hiranyagarbha, the golden germ (whether the original of that name may have been), or whom the poet says - In the beginning there arose Hiranyagarbha; he was the one born lord of all this. He established the earth and this sky. Who is the god to whom we shall offer our sacrifice?. That Hiranyagarbha, the poet says, is alone God above all gods - an assertion of the unity of the Divine which could hardly be exceeded in strength by any passage from the Old Testament."

It is quite clear that Western researchers are confused. This confusion arises out of ignorance towards correct method of decoding the hymns. This key to unlock the treasure is provided by Yāska (यास्क) in Nirukta (निरुक्त).

3.2.2 Classification provided by Yāska

To help researchers understand the Vedic deities better and to help them realize their areas of operation and scope, Yāska provides crystal clear directives. Interestingly, this clarity provided by Yāska (यास्क) is based on one Mantra (1.139.11) from Ṛgveda (ऋग्वेद). The hymn is as follows:

- ये देवासो दिव्येकादश स्थ पृथिव्यामध्येकादश स्थ ।
  अप्सुक्षितो महिनैकादश स्थ ते देवा यज्ञमिमं जुषध्वम् ॥ ऋग्वेद १.१३९.११

Mantra (1.139.11) from Ṛgveda (ऋग्वेद) clearly describes 11 deities each in Pṛthvī (पृथ्वी), Antarikṣa (अन्तरिक्ष) and Dyuaḥ (द्युः).

R There are only three primeval deities and others are only variants of these three. These three primary deities are:
- Agni (अग्नि) on Pṛthvī (पृथ्वी)
- Vāyu (वायु) or Indra (इन्द्र) in Antarikṣa (अन्तरिक्ष)
- Sūrya (सूर्य) in Dyuaḥ (द्युः) or Svarga (स्वर्ग)

There is enough room for imagination to also think of these three worlds on temporal axis and that will help put many pieces in the jigsaw puzzle. The imaginary picture of this can be as depicted as in Figure (3.1):

Prof. Macdonnel has listed these corresponding deities in the three worlds as follows: [[63]]

- 11 deities in Svarga (स्वर्ग) - Dyauḥ (द्यौः), Varuṇa (वरुण), Mitra (मित्र), Sūrya (सूर्य), Savitṛ (सवितृ), Pūṣan (पूषन्), Viṣṇu (विष्णु), Vivasvat (विवस्वत्), Ādityagaṇa (आदित्यगण), Uṣas (उषस्), Aśvina-Yugala (अश्विन - युगल)
- 11 deities in Antarikṣa (अन्तरिक्ष) - Indra (इन्द्र), Trita (त्रित), Āptya (आप्त्य), Apānapāt (अपानपात्), Mātariśvan (मातरिश्वन्), Ahirbudhnya (अहिर्बुध्न्य), Aja Ekapāda (अज एकपाद), Rudra (रुद्र), Marudgaṇa (मरुद्गण), Parjanya (पर्जन्य), Āpaḥ (आपः)
- 11 deities in Pṛthvī (पृथ्वी) - Rivers (नद्या), Pṛthivī (पृथिवी), Agni (अग्नि), Soma (सोम)

The deities are enlisted, however it is important to comprehend the nature of these three worlds. Only then, we will be able to assign the hymns to various aspects of the three worlds and derive meaningful interpretation. The classical characterization of these three worlds in Yāska (यास्क) style is as depicted in Table (3.1).

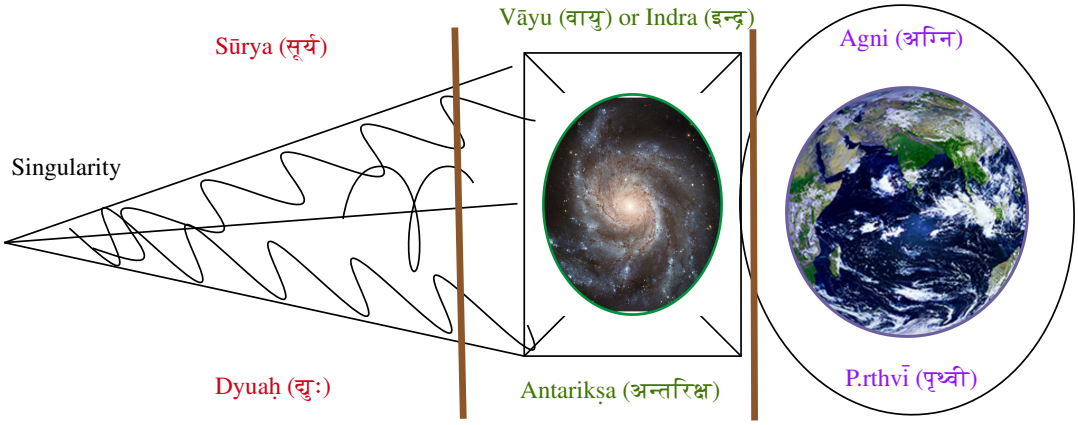


Figure 3.1: Three Worlds as per Yāska

It is interesting to note that the hymns expressed in Chanda (छन्द) or meter Triṣṭup (त्रिष्टुप्) predominantly represent the happenings in Antarikṣa (अन्तरिक्ष). The present Nāsadīya Sūkta (नासदीय सूक्त) however, must be an exception to this. We should take into consideration that this hymn is the hymn of creation and hence goes beyond the scope of one particular world.

The concept is extremely useful though! For example, Indra (इन्द्र) is in human form being the King of Gods on Pārthiva (पार्थिव) world, and it is purely of energy form in the remaining two worlds. The other forms will be uncovered and explained in detail in later part of this thesis, however at this point it is important to note that the nature and manifestation is different in these different worlds. Since this important link was overlooked or not understood by most of the western scholars, their interpretations are skewed and biased. They either put their brains to map the entities strictly on physical characteristics or completely in spiritual or mythological or imaginative realms. The appropriate understanding has a main connect in figuring out which world the hymn and the verse belongs to and then interpret it accordingly. This was very well comprehended by Sāyaṇācārya (सायणाचार्य), however it was so obvious amongst Indian scholars, that he has not mentioned it explicitly anywhere. That could be the reason why, western scholars particularly - without any exception, missed this important link in the chain.

In this research work, we intend to establish the correspondence between the Vedic science and the modern science. Therefore, it is important to understand these three worlds and their corresponding realms in accordance with the modern science. The detailed explanation of this will get reflected at multiple instances in this thesis, however the fundamental depiction is brought out for clarity and demystification. As per hymn (5.69.1/2/3/4) of Ṛgveda (ऋग्वेद), the scientific characterization of these three worlds is as shown in Table (3.2). This is in line with the modern understanding of physical world, comprising the earth, the space and the construct of galaxies.

Readers should note here that, Gravitational Force (गुरूत्वाकर्षण) operates only at Pārthiva (पार्थिव) level and only Pārthiva (पार्थिव) and Antarikṣa (अन्तरिक्ष) are bound by the time! This is extremely important in deciding the chronological sequence of creation of the universe (विश्वोत्पत्ति).

World (जगत्रय)	Dyuḥ (द्युः)	Antarikṣa (अन्तरिक्ष)	Pārthiva (पार्थिव)
Deity (देवता)	Āditya (आदित्य)	Vāyu/Indra (वायु वा इन्द्र)	Agni (अग्नि)
Sthāna (स्थान)	Svaḥ (स्वः)	Bhuvaḥ (भुवः)	Bhuḥ (भुः)
Savana (सवन)	Sāyaṃ (सायं)	Mādhyāndina (माध्यान्दिन)	Prātaḥkāla (प्रातःकाल)
Ṛtū (ऋतू)	Varṣā (वर्षा)	Grīṣma (ग्रीष्म)	Vasanta (वसन्त)
Chanda (छन्द)	Jagatī (जगती)	Triṣṭupa (त्रिष्टुप)	Gāyatri (गायत्रि)
Prārthan (प्रार्थन्)	17 (१७)	15 (१५)	3 (३)
Karma (कर्म)	ĀdityaKarma = Pravahlitam (आदित्यकर्म = प्रवह्लितम्)	IndraKarma = BalaKṛtiḥ (इन्द्रकर्म = बलकृतिः)	AgniKarma = Dārṣṭiviṣayakam (अग्निकर्म = दार्ष्टिविषयकम्)
Adhika Ṛtū (अधिक ऋतू)	Śiśira (शिशिर)	Hemant (हेमन्त)	Śarada (शरद)
Adhika Chanda (अधिक छन्द)	Atichandā (अतिछन्दा)	Paṅkti (पंक्ति)	Anuṣṭupa (अनुष्टुप)
Stoma (स्तोम)	33 (३३)	27 (3x9) (२७)	21 (२१)

Table 3.1: Three Worlds - Philosophical

World (जगत्रय)	Dyuḥ (द्युः)	Antarikṣa (अन्तरिक्ष)	Pārthiva (पार्थिव)
Sthāna (स्थान)	Svaḥ (स्वः)	Bhuvaḥ (भुवः)	Bhuḥ (भुः)
Wave (Vāṇī) (वाणी)	Parā (परा)	Paśyanti (पश्यन्ति)	Madhyamā + Vaikharī (मध्यमा + वैखरी)
Form (स्वरूप)	Sat (सत्)	Seed (Sūkṣma) (सूक्ष्म)	Gross (Sthūla) (स्थूल)
Gravitational Force (गुरूत्वाकर्षण)	☐ No (नाही)	☐ No (नाही)	☑ Yes (हो)
Time (काळ)	☐ No (नाही)	☑ Yes (हो)	☑ Yes (हो)

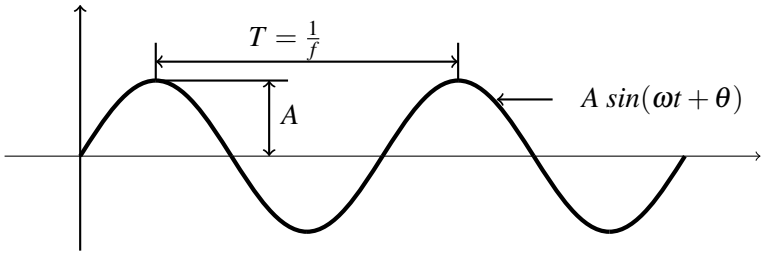
Table 3.2: Three Worlds - Scientific

3.2.3 Energy Analysis Fundamentals

It is interesting and important to note that all the three worlds (जगत्त्रय) as depicted by Yāska are of the energy form. The energy gets transformed while moving from one world to another and hence now we present the energy based interpretation of the three worlds. For this we will revisit few important concepts from the modern signal processing. This will not only help us understand the nature of these three worlds, but it will also enable us to decipher the very reason for the universe to come into existence as stated in Ṛcā (ऋचा) four of Nāsadīya Sūkta (नासदीय सूक्त).

Let's examine the most natural form of signals that exist in the world, these are continuous signals.

§ Continuous Signals:



It is to be noted that the given signal is expressed mathematically as $A\sin(\omega t + \theta)$. Here A is the amplitude or the magnitude of the signal, ω represents the frequency and θ is the phase. The given signal is function of time (काळ) t.

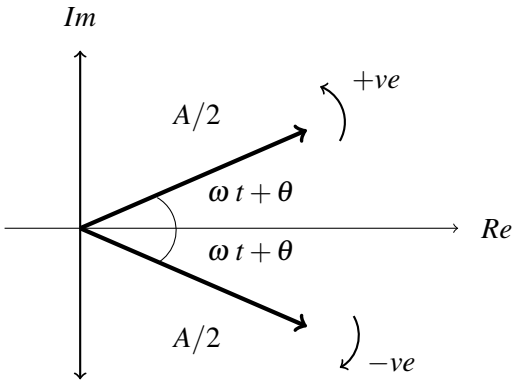
The same signal can be expressed as phasor for the ease of computing using 'Euler's Identity'.

The derived version of Euler's Identity is as follows:

$$e^{\pm j\phi} = cos(\phi) \pm jsin(\phi) \tag{3.1}$$

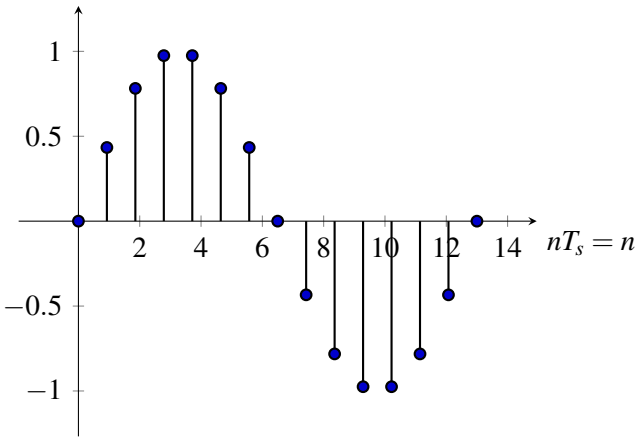
$$\text{if, x(t)} = A\,cos(\omega\, t + \theta) \tag{3.2}$$

$$= \frac{A}{2}\, e^{j\,(\omega\, t+\theta)} + \frac{A}{2}\, e^{-j\,(\omega\, t+\theta)} \tag{3.3}$$



Since, the signal is considered as combination of rotating complex exponential with opposite angular frequency, the phase mapping becomes much simpler. In the modern era of powerful computing, we typically use the discrete (after sampling) and further digital (after quantization) form of the continuous signals.

§ Discrete Signals:



$$x(t) = A\,sin(\omega t) \tag{3.4}$$
$$= A\,sin(2\pi f t) \tag{3.5}$$

$$x(n) = A\,sin(2\pi f \cdot nt_s) \tag{3.6}$$
$$= A\,sin(2\pi f \cdot n \cdot \frac{1}{f_s}) \tag{3.7}$$
$$= A\,sin(2\pi n \cdot \frac{f}{f_s}) \tag{3.8}$$
$$= A\,sin(2\pi F n), F = \frac{f}{f_s} \quad \frac{\rightarrow i/p}{\rightarrow sampling} \tag{3.9}$$
$$\therefore \quad x[n] = A\,sin[\omega n]\,, \quad \omega = 2\pi f \tag{3.10}$$
$$x[n] = A\,sin[\omega n + \theta] \tag{3.11}$$

Thus, $x[n]$ here is a discrete signal.

These variations are only for the convenience. Deeper understanding of the signals in general, emerges out of bringing out understanding of whether the signal is a 'Power signal' or an 'Energy Signal'. The salient feature of these types of signals is as under:

- A signal is referred as energy signal if $0 < E < \infty$
- Power signal : $0 < P < \infty$
- Energy signal : '0' average power $\rightarrow$ Deterministic & non-periodic.
- Power signal : '∞' energy $\rightarrow$ random & periodic.

The total energy in a given continuous signal $x(t)$ is given as:

$$E_C = \int_{-\infty}^{\infty} x^2(t)dt \tag{3.12}$$

For discrete signal $x[n]$, the energy is calculated as:

$$E_D = \sum_{-\infty}^{\infty} x^2[n] \tag{3.13}$$

Similarly, let's examine the formulae for calculating the power in signals. The power in continuous signal $x(t)$ with fundamental period T is given as:

$$P_C = \frac{1}{T}\int_{-T/2}^{T/2} x^2(t)dt \tag{3.14}$$

For discrete signal $x[n]$ with fundamental period N, the power is calculated as:

$$P_D = \frac{1}{N}\sum_{0}^{N-1} x^2[n] \tag{3.15}$$

The summary of this can be represented in Table (3.3) below:

Type of Signal	Power	Energy
Vedic Name	Śakti (शक्ति)	Ūrjā (ऊर्जा)
Continuous Domain	$P_C = \frac{1}{T}\int_{-T/2}^{T/2} x^2(t)dt$	$E_C = \int_{-\infty}^{\infty} x^2(t)dt$
Discrete Domain	$P_D = \frac{1}{N}\sum_{0}^{N-1} x^2[n]$	$E_D = \sum_{-\infty}^{\infty} x^2[n]$
Energy	$P = 0$	$0 < E < \infty$
Power	$0 < P < \infty$	$E = \infty$
Nature	Periodic and Stochastic	Aperiodic and Deterministic

Table 3.3: Power and Energy Signals

3.2.4 More on Energy Analytics

It will be shown that Power signals are not meant for transmission of any kind. This physical world is essentially governed by the 'Energy Signals'. For periodic power signals, the analysis is performed by using series expansion methods. There are many known series expansions like Taylor series, Fourier Series, Maclaurin Series to name a few. For aperiodic energy signals however, 'Transform' techniques are needed, with the likes of Fourier Transform, Laplace Transform, Z-Transform to name a few.

Need for transformations

There are many reasons for carrying out a transformation, most significant one is convenience! For example we do not understand music as just few time domain signals with varying voltage, we understand music as sequence of frequencies. Thus it makes sense to transform musical signal into frequency domain and then the analyzer will be more comfortable dealing with those frequencies. Ultimately we want to design filters, more pertinently filter banks in the context of wavelets, and it is more convenient to design filter banks in frequency domain compared to time domain or spatial domain.

For all conventional transforms like Laplace transform, Z transform, Fourier Transform and Logarithmic Transform the basis function comes from natural logarithmic base e. However for understanding wavelet transform we have to go beyond purview of these conventional transforms. We already know how to analyze Linear Time Invariant (LTI) Systems. For doing so traditionally we have two methods,

1. Convolution
2. Difference Equations

Convolution comes out of the fact that given a signal x[n], we can decompose that signal into scaled and shifted impulse sequences. After doing so we can write,

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k] \tag{3.16}$$

This is the scaled summation of shifted impulses. We are able to do this because the system is assumed to be linear and shift invariant. Since the system is linear and thus would follow superposition theorem we are able to add up the products to get x[n]. Also because of time invariance property we are able to shift the impulses without affecting the end result.
Once we do this and understand how the system reacts to impulses as stimulus, we would be able to get the systems impulse response. Now if we call the impulse response as h[n], excite the LTI system with an exponential $e^{j\omega_0 n}$ as an input x[n] and call the output as y[n] we would have,

$$y[n] = \sum_{-\infty}^{\infty} e^{j\omega_0 n} h[n-k] \tag{3.17}$$

However we know by commutative law,

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] = \sum_{-\infty}^{\infty} x[n-k]h[n] \tag{3.18}$$

Thus by putting $e^{j\omega_0 n}$ in place of x[n] we have,

$$y[n] = \sum_{k=-\infty}^{\infty} e^{j\omega_0[n-k]} h[n] \tag{3.19}$$

This forms an interesting eigen system for LTI analysis. That is because we can split the exponential term from equation (3.19) and then the summation happens for variable k and the term $e^{j\omega_0 n}$ can be brought out out of this summation of k terms.

$$y[n] = \sum_{k=-\infty}^{\infty} e^{j\omega_0[n]} e^{j\omega_0[-k]} h[n] \tag{3.20}$$

$$y[n] = e^{j\omega_0[n]} \sum_{k=-\infty}^{\infty} e^{j\omega_0[-k]} h[n] \tag{3.21}$$

From equation (3.21) we can see that after exciting the system with exponential of frequency ω_0 the result is the same term with something. The term $e^{j\omega_0 n}$ is known to us as eigen function and the multiplying term is known as a eigen value. This eigen value in a broader sense is known to us as a Fourier Transform. The basis function here is exponential e. This fact as discussed earlier in this chapter remains the same for other transforms also, for example in Z transform the basis function Z also equals $e^{j\omega}$. It would be interesting now to look at from where this constant 'e' comes from.

The constant e was discovered by Dr. Bernoulli in a vague accident for which the story doesn't have authentic source, it is still very interesting though. Dr. Bernoulli was trying to help his banker

friend because his business was not picking up. He gave him a solution based on the formulae for calculation of compound interest. As we know the formula for investment of a one rupee or one dollar or one euro as principle amount is given as,

$$\{1+\frac{1}{n}\}^n$$

Dr. Bernoulli brought in series expansion by applying limit for n terms to infinity,

$$\lim_{n\to\infty}\{1+\frac{1}{n}\}^n \tag{3.22}$$

and the constant 'e' came into existence, waiting for Euler to give it meaning! An engineer's perspective is provided in the MATLAB code below for readers to try out the above limit approximation through simulations (rather than solving it actually – may be a mathematician's perspective!) and get convinced that it indeed gives us the constant after few iterations.

Analytics 3.2.1 — MATLAB code to approximate constant 'e'. \\

```
%***********************************************
%***********************************************
% MATLAB code to understand calculations for
% Constant 'e': To be accompanied with thesis by
% by Aditya Abhyankar
%***********************************************
%***********************************************
clear all;close all;clc;

for n=1:1000000000  % MATLAB doesn't support zero index
                    % we can't take 'n' all the way to infinity
                    % hence some big number 1000000000
    (1+(1/n))^n     % compound interest formulae
    pause;          % to see the value after every iteration
                    % slowly value saturates to 2.7183
end
% End
```

acob Bernoulli

Jacob Bernoulli (1654-1705) was son of Nicolas Bernoulli and elder brother of Johann Bernoulli. Jacob acquired degree in theology to start his formal education as his father was against his children taking up mathematics. In spite of that due to his deep interest and curiosity he became mathematician. In 1687 he became Professor of mathematics at University of Basel, where he

taught till the end of his life. He continued working on the lines of Napier and was able to successfully discover the constant 'e'. His contributions in the field of calculus are also well known and respected.

Though the story of banker friend of Dr. Bernoulli lacks authenticity, it was certain that around 1889 he published series of articles on theory of infinite series and proposed continuously compounded interest. There were also some series expansions like

$$1+\frac{1}{4}+\frac{1}{9}+...+\frac{1}{k^2}+...$$

for which he couldn't settle with a converging number and the quest was completed by Dr. Euler when he showed that above infinite summation equates to $\pi^2/6$. Thus in many ways Dr. Euler took Dr. Bernoulli's work forward. After Dr. Bernoulli invented constant e, Dr. Euler came up with an identity known today as Euler's Identity which is still regarded as one of the most beautiful of all mathematical discoveries.

$$e^{i\pi} = -1 \tag{3.23}$$

From application perspective more useful version of Euler's Identity is,

$$e^{i\theta} = cos(\theta) + jsin(\theta) \tag{3.24}$$

The interesting thing to note is if we draw a tangent to any point on this exponential curve the y-intercept and slope of this tangent matches. Thus at any point along the curve we can resolve the point into 'cos' and 'sin' components which are in turn orthogonal in nature. Dr. Euler really gave special meaning to this constant 'e' and it's because of his contribution we get orthogonal and energy preserving transforms. It's because of this unique property of 'e', the inverse of the transform also exists.


eonard Euler

Leonard Euler (1707-1783) was a Swiss mathematician and he was well known for the quality as well as quantity of his work. His contributions in the field of graph theory, number theory and calculus are considered to be outstanding. Born in Basel in 1707, in 1727 he went to St Petersburgh Academy. In 1741 he joined Berlin Academy and in 1766 joined back St Petersburgh Academy. He died in 1783. In 1748 he gave a meaning to the constant 'e' and gave base to the natural logarithm which then went on to become basis to most of the transforms.

After Dr. Euler gave a powerful meaning to 'e', Dr. Fourier worked on analyzing periodic and aperiodic functions and signals. The legacy of transforms is thus result of contribution from these

three great scientists. Dr. Fourier gave a systematic way of analyzing periodic as well as aperiodic signals through Fourier series and Fourier Transform framework. For a time domain signal $x(t)$ the corresponding Fourier Transform is given by following equation:

$$\hat{X}(\Omega) = \int_{-\infty}^{\infty} x(t) e^{-j\Omega t} dt \tag{3.25}$$

Thus, conceptually Fourier description is the inner product between the signal to be analyzed and the basis or kernel function.

acob Fourier

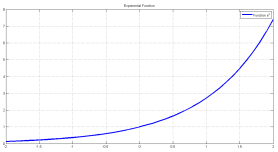
(1768 1830) Jacob Fourier was a French mathematician and a well known scientist with the contributions like Fourier series, Fourier descriptors, Fourier transform etc. Fourier introduced the notion of representing continuous as well as discontinuous functions using continuous basis functions. This was an important and path-breaking research which also encouraged scientists like Laplace to build their own theories like Laplace transform.

Now, let's look at question why wavelet transform? The wavelet transform decomposes signal into two separate series. One which represents the coarse version which leads to scaling function and other which represents the details or the refined version which leads to wavelet function or the mother wavelet function.
Still two more questions need to be answered, aren't the traditional methods of representing signals good enough? And what is so special about this wavelet transform? We will answer this gradually in this book.
Lets take these questions one at a time. In the traditional methods the most basic representation of signals is with Taylor series. For e.g. Taylor series expansion at $x_0 = 0$,

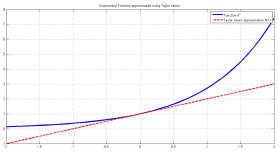
$$e^x = 1 + \frac{x^1}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots = \sum_{n=0}^{\infty} \frac{x^n}{n!} \tag{3.26}$$

Every single coefficient in the above expression can be looked upon as a decomposed piece and such pieces can be used for reconstructing the corresponding signal or function. If we only make use of finite number of coefficients lets say up to first 5 coefficients and try to reconstruct the signal then the reconstructed signal would look like the signal shown in figure (3.2).

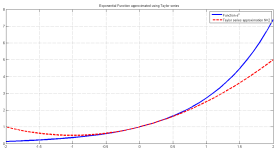
In the figure (3.2) sub-figures (3.2b),(3.2c),(3.2d),(3.2e), (3.2f) show Taylor series approximation of exponential function with number of coefficients used to be $N = 1$,$N = 2$,$N = 3$,$N = 4$, and $N = 5$ respectively. It is clear that as the number of coefficients increase the approximation (shown as red dotted line) starts approaching the actual function (shown as blue solid line). Thus in Taylor series, cooperation to build better representation is rigid from the perspective of lack of inter-coefficient resolution. Since we have to work with large terms, also scale and translation of every single term is limited. In contrast to this in wavelet analysis the scaling function and its associated wavelet function makes the representation flexible.



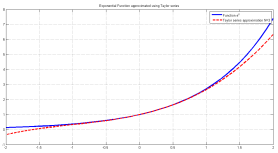
(a) Exponential Function $y = f(x) = e^x$



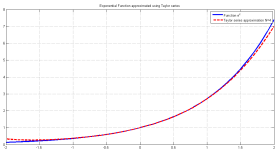
(b) Taylor series approximation using $N = 1$



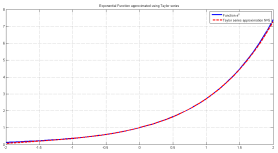
(c) Taylor series approximation using $N = 2$



(d) Taylor series approximation using $N = 3$



(e) Taylor series approximation using $N = 4$



(f) Taylor series approximation using $N = 5$

Figure 3.2: Approximation of exponential function using up to 5 coefficients from Taylor series.

The MATLAB code to try more coefficients out is provided for readers:

Analytics 3.2.2 — MATLAB code to understand Fourier Transform. \\

```
%************************************************
%************************************************
% MATLAB code to understand working of Fourier
% Transform: To be accompanied with thesis by
% by Aditya Abhyankar
%************************************************
%************************************************
clear all;close all;clc;
x=-2:0.1:2; % Create x-axis
y=exp(x);   % The exponential function y=e^x
figure(1);
plot(x,y,'b','LineWidth',3); % The function in blue
grid on; title('Exponential Function approximated using Taylor
series');
N=5;    % No of coeffs in Taylor series approximation
y_est=0;% Initialize the estimate to zero
for n=0:N
    y_est=y_est + (x.^n)./factorial(n); % Taylor series formulae
end hold on;
plot(x,y_est,'--r','LineWidth',3) % Plot the approximation as red dotted
legend('Function e^{x}','Taylor series approximation N=5') % legend
% End
```

This is useful, for example in wavelet analysis the scale $\{1/2\}^j$ is dependent upon the analyzer. Thus while sampling a high frequency signal we can bring in a very high value of j. Then a translation $\tau_{j,k} = k/2^j$ can be used to focus on that part. Hence with wavelet analysis we can look into any

particular part of the signal. By changing the scale and translation parameter we have an interesting zoom-in or zoom-out facility of wavelet transform. Fourier series has a noteworthy advancement over the Taylor series. Since elements of a Taylor series do not necessarily and always form an orthogonal set. However in case of Fourier series the set $\{1, cos(nx), sin(nx)\}_{n=1}^{\infty}$ is always orthogonal on $(-\pi, \pi)$. But still flexibility with scale and translation parameters remains to be explored.

Even though Fourier series is rigid in terms of rotations and translations, we derive some information related to wavelet transformation from it. Fourier series is represented by following equation,

$$f(x) = a_0 + \sum_{k=1}^{\infty} [a_k cos(kx) + b_k sin(kx)] \tag{3.27}$$

Where,

$$a_0 = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(x)dx$$
$$a_k = \frac{1}{\pi} \int_{-\infty}^{\infty} f(x)sin(kx)dx$$
$$b_k = \frac{1}{\pi} \int_{-\infty}^{\infty} f(x)cos(kx)dx$$

We observe that a special relationship exists between cosine and sine terms of the series. A similar relationship also exists between scaling $\phi(.)$ and wavelet $\psi(.)$ functions. This relationship although trivial is very interesting. Though wavelets are radically different than conventional transforms like Fourier transform, its rise has its roots in the shortcomings of these conventional transforms. This concept is useful in establishing the 'Advait' (अद्वैत) philosophy at the root of the creation of the universe.

With this background, we have enough tools to start the journey towards uncovering the Nāsadīya Sūkta.

3.3 Nāsadīya Sūkta - Deciphering the Hymn

The following sub-sections discuss every verse independently now.

3.3.1 Nāsadīya Sūkta - Verse Number 1

The very first verse along with the anvaya (अन्वय) is as follows:

.. Ṛk No 1 ..

.. ऋक् क्र. १ ..

- नासदासीन्नो सदासीत्तदानीं नासीद्रजो नो व्योमा परो यत् ।
  किमावरीवः कुह कस्य शर्मन्नम्भः किमासीद्गहनं गभीरम् ॥ १०.१२९.०१

- अन्वय - न । असत् । आसीत । नो इति । सत् । आसीत । तदानीं । आसीत । रजः । नो इति । विऽओम । परः । यत् । किं । आ । अवरीवरिति । कुह । कस्य । शर्मन् । अंभः । किं । आसीत् । गहनं । गभीरं ॥ १०-१२९-०१
- शब्दार्थ - न = no । असत् = non-being or non-existent । आसीत = existed । नो इति = also not । सत् = being or existent । आसीत = existed । तदानीं = Before universe । आसीत = existed । रजः = middle of the three Guṇas or innate qualities, leading to action-passion । नो इति = also not । विऽओम = encompassing sky । परः । यत् = beyond । किं । आ । अवरीवरिति = Who covered whom? । कुह = Where? । कस्य । शर्मन् = For what purpose(pleasure) । अंभः । किं । आसीत् । गहनं । गभीरं ॥ = What that deep and infinite water was? १०-१२९-०१

For every hymn we take into consideration commentaries provided by 15 scholars across the globe. These scholars are from vivid religious, geographical, cultural backgrounds and everyone adds value to our understanding. It should be mentioned however, the interpretations given by Swamī Vivekānanda, Sāyaṇācārya and others who follow them are closest to our heart. The interpretations of these scholars are provided as a ready reference before giving our decoding of the hymn. Following Table (3.4) provides the views and interpretations of these 15 scholars on verse 1 of the present hymn. Author has also looked as the interpretations presented by Dr. Bheemarao S Kulkarni [24] and Shri. R G Kolangade [25]. Both these works are very profound and scholarly, for the present hymn however, they closely follow Citrāva Śāstrī (चित्राव शास्त्री) [38] and hence are not reproduced as ready reference. Author would also like to mention a prudent and comprehensive and yet lucid work by Shripad Damodar Satwalekar [64] with gratitude.

Author would like to state that the work done by Āhitāgni Rājawāḍe (आहिताग्नि राजवाडे) is unique and author has drawn inspiration from few concepts from this work [26]. Authors would also like to state that it is indeed disheartening to observe strong hatred in the writings of Āhitāgni Rājawāḍe (आहिताग्नि राजवाडे) towards the work done and theories proposed by Ādi Śaṅkarācārya (आदि शंकराचार्य) and authors strongly condemn the same. In fact, authors would like to express deep gratitude towards Ādi Śaṅkarācārya (आदि शंकराचार्य) and in this thesis it is shown that various theories ultimately converge into Advaita (अद्वैत) philosophy. Few interesting but less accurate interpretations like [65] and [66] are not taken into consideration.

We will evaluate all the verses at three different levels:

1. At spiritual conceptual level
2. At physical level
3. At Temporal level

This is the reason, this study is tri-fold study.

Analytics 3.3.1 — Proposed decoding of Verse 1. We will now look at every word and try and decipher it towards one unified interpretation. The attempt is to blend the spiritual view with the scientific view and show that they have huge overlap and commonalities.

तदानीं**:-** The word 'Tadānīṃ' (तदानीं) indicates the time when the universe did not exist. Thus, this sūkta (सूक्त) starts the discussion from the times prior to the genesis of the universe. This is a confirmatory test that this sūkta (सूक्त) discussed about the origin of the universe. The very first verse captures the essence of creation of the universe. The theorem is expressed in a subtle way and needs to be understood carefully.

Comparative Interpretations of scholars on Verse 1 of Nāsadīya Sūkta	
Name of the interpreter	Interpretation
Vivekananda [27]	Existence was not then, nor non-existence, The world was not, the sky beyond was neither. What covered the mist? Of whom was that?What was in the depths of darkness thick?
Max Mueller [28]	There was then neither what is nor what is not, there was no sky, nor the heaven which is beyond. What covered? Where was it, and in whose shelter? Was the water the deep abyss (in which it lay)?
A. A. Macdonell [29]	Non-being then existed not nor being: There was no air, nor sky that is beyond it. What was concealed? Wherein? In whose protection? And was there deep unfathomable water?
Raimundo Panikkar [30]	At first was neither Being nor Nonbeing. There was not air nor yet sky beyond. What was wrapping? Where? In whose protection? Was Water there, unfathomable deep?
Griffith [31]	THEN was not non-existent nor existent: there was no realm of air, no sky beyond it. What covered in, and where? and what gave shelter? Was water there, unfathomed depth of water?
Jamison & Brereton [32]	The non-existent did not exist, nor did the existent exist at that time. There existed neither the airy space nor heaven beyond. What moved back and forth? From where and in whose protection? Did water exist, a deep depth?
Geldner [33]	Weder Nichtsein noch Sein war damals; nicht war der Luftraum noch der Himmel darber. Was strich hin und her? Wo? In wessen Obhut? Was war das unergrndliche tiefe Wasser?
A. L. Basham [34]	Then even nothingness was not, nor existence. There was no air then, nor the heavens beyond it. What covered it? Where was it? In whose keeping? Was there then cosmic water, in depths unfathomed?
Wilson, HH: [35]	The non-existent was not, the existence was not; then the world was not, not the firmament, nor that which is above (the firmament). How could there be any investing envelope, and where? Of what (could there be) felicity? How (could there be) the deep unfathomable water?
Wendy Doniger O'Flaherty [36]	There was neither non-existence nor existence then. There was neither the realm of space nor the sky which is beyond. What stirred? Where? In whose protection? Was there water, bottlemlessly deep?
Pandit RamGovind Trivedi [37]	उस समय या प्रलय दशा मे असत् (सियार की सींग की समान जिसका अस्तित्व नही है) नही था। जो सत् (जीवात्मा आदि) है, वह भी नही था। पृथिवी भी नही थी और आकाश तथा आकाश मे विद्यमान सतो भुवन भी नही थे। आवरण (ब्रह्माण्ड) भी कहा था? किसका कहा स्थान था? क्या दुर्गम और गम्भीर जल उस समय था?
Siddheshwar Shastri Chitrav [38]	सृष्टीच्या मूलारंभी असत् आणि सत्, तसेच अंतरिक्ष आणि आकाश यापैकी काहीच अस्तित्वात नव्हते. अशा स्थितीत कोणी कोणाला आवरण घातले? कोणाच्या सुखासाथी हे सारे झाले? अगाध जल तरी त्या काली अस्तित्वात होते काय?
Krishnananda [39]	Then even nothingness was not, nor existence, There was no air then, nor the heavens beyond it. What covered it? Where was it? In whose keeping Was there then cosmic water, in depths unfathomed?
Hari Damodar Velankar [40]	उस समय (याने सृष्टिके पूर्व) यह सत् भी नही था, असत् भी नही था । तब अन्तरिक्ष लोक या उसके पार रहनेवाला जो आकाश वह भी नही था । (तो फिर) उसने (अपने उदरमे) क्या छिपा रखा था? वह कहा और किसकी सुरक्षा मे था? अगाध और गहरा, ऐसा जल तो उस समय क्या विद्यमान भी था?
Sāyaṇācārya [41]	तपसस्तन्महिनाजायतैकम् इत्यादिनाग्रे सृष्टिः प्रतिपादयिष्यते। अधुना .....

Table 3.4: Contemporary Interpretations: Verse 1 of Nāsadīya Sūkta

न असत् आसीत् नो सत् आसीत्:- Neither was there 'Asat' (असत् = not being), nor was there 'Sat' (सत् = being)!

> R The most fundamental form of Dvandva (द्वंद्व), expressed by 'Sat-Asat' (eternal-ephemeral) or 'Asti-Nāsti' (being-not being), is the genesis of the beginning of the physical world. Here 'Sat' suggest existence and 'Asat' indicates absence. This captures in the very essence of symbolically mapping the beginning of the world! We will also show that this Dvandva (द्वंद्व) eventually converges into Dvaita (द्वैत).

रजः:- Now, let's examine रजः. There are two ways to look at it:

1. Rajaḥ (रजः) = middle of the three Guṇas or innate qualities, leading to action-passion. It may appear strange that there is no mention of either Satva (सत्व) or Tamo-Guṇa (तमो - गुण), however it is logical.
2. Rajaḥ (रजः) = Rajoloka (रजोलोक). This is the middle one in Pṛthivī (पृथिवी) - Antarikṣa (अन्तरिक्ष) - Dyu (द्यु), that is Antarikṣa (अन्तरिक्ष). This is in line with this hymn being in Triṣṭupa (त्रिष्टुप). It could also mean, the middle one in Bhūḥ (भूः) - Bhuvaḥ (भुवः) - Svaḥ (स्वः), that is Bhuvaḥ(भुवः).

For both the cases, the choice of is befitting and apt. The word 'Rajas' (रजस्) germinates from root रंज् = रंगणे, which means to be colored. Thus, 'Rajas' indicates varied colors - the colors being many - it also indicates plurality. The big picture associated with the use of word 'Rajas' is depicted in Figure (3.3). 'Rajas' (रजस्) indicates plurality as it represents multifold colors. 'Sattva' (सत्त्व) can be depicted with white color, where white essentially reflects every wavelength (λ) back. 'Tama' gets represented with Black color, as it essentially absorbs wavelengths (λ) corresponding to all the colors. From the reference of the color bands, thus 'Satva' and 'Tama' form yet another 'Dvandva' (द्वन्द्व = contradictory pair). Thus, this physical world (जगत) on the Pārthiva (पार्थिव) has two salient features:

- By mixture of all the three Guṇas (त्रिगुण), it is 'Triguṇānvita' (त्रिगुणान्वित)
- It is made up of contradictory pairs and hence is 'Dvandvamaya' (द्वन्द्वमय)

The physical world hence is plural, partly manifested and partly not manifested (अनेकरूप , व्यक्ताव्यक्त). Satva-Guṇa leads to 'Sat' (सत्) which is: singular, non-unique and manifested (एकरूप , सद्वितीय , व्यक्त). Tama-Guṇa leads to 'Asat' (असत्) which is: singular, non-unique but not manifested (एकरूप , सद्वितीय , अव्यक्त). The creator (जगदीश) is beyond the reals of the physical world (जगदातीत), singular (एकमेवाद्वितीय), Unified (निर्द्वंद्व) and beyond the three Guṇas (निस्त्रैर्गुण्य). This holds true for Pārthiva world. For Antarikṣa, it is Advaita (अद्वैत) and for Dyuḥ it is Advaya (अद्वय)!

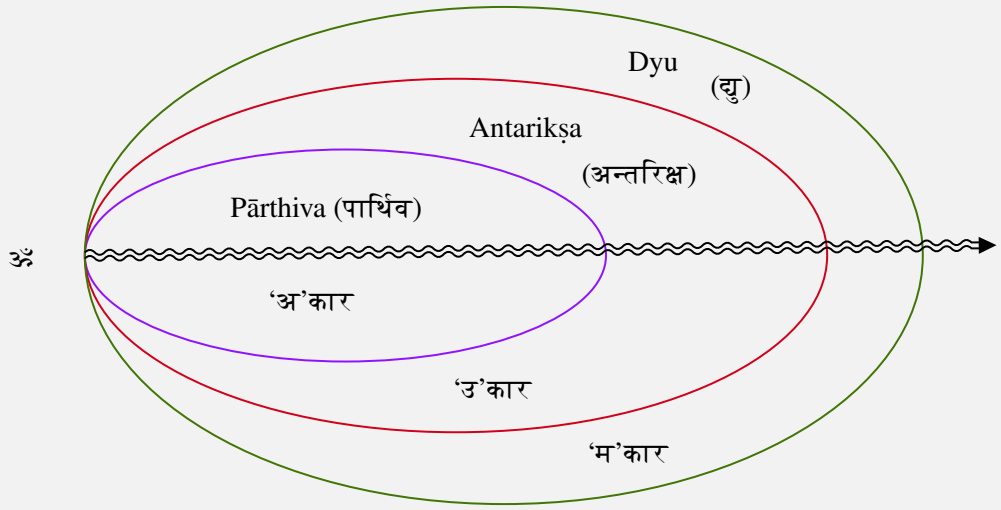
Sāyaṇācārya (सायणाचार्य) has interpreted Rajas (रजस्) as 'Rajo-Guṇa' (रजो - गुण) in Taittirīya Bhāṣya (तैत्तिरीय भाष्य) and as 'Rajo-Loka' (रजो - लोक) in Ṛgbhāṣya (ऋग्भाष्य) [[41]]. From the above discussion it is clear however, that it does not make any difference as what we are discussing is beyond the realms of Lokas and also Guṇas. Interesting enough, it also provides the link for the next exploration. Rajas (रजस्) also leads to Rajaḥkaṇa (रजःकण), which are dust particles. Fort these particles to reside and propagate, they need some medium. Typically this medium is void of some kind which is indicated next.

We conclude, that 'Rajas' essentially indicates the Antarikṣa (अन्तरिक्ष) which also did not exist at pre-primordial times.

परः विऽओमः:- Here Paraḥ (परः) means beyond. This is connected with 'Rajas' indicating Antarikṣa (अन्तरिक्ष). Therefore, 'Vyoman' (व्योमन्) indicates the third world, that is Svar-

loka (स्वर्लोक) or Dyuloka (द्युलोक). This is AEtherial region which consists of multiple galaxies. Etymologically Yāska has given definition of Antarikṣa (अन्तरिक्ष) as 'अंतरा द्यावापृथिव्याःक्षांतं अवस्थितं भवति'. Thus, 'Vyoman' (व्योमन्) indeed indicates Dyuloka (द्युलोक). At pre-primordial times, even Dyuloka (द्युलोक) did not exist.

किं कुह आवरीवः कस्य शर्मन् ?:- Who covered whom? Where was this cover done? For whose existence was this cover done? The word Āvarīvaḥ (आवरीवः) is the key here. This description clearly states that Dyuloka (द्युलोक) is the super-set, Antarikṣa (अन्तरिक्ष) is the sub-set and Pārthiva (पार्थिव) is the sub-sub-set. This hierarchy of the three worlds is neatly captured by this construct. The same is depicted below:



गहनं गभीरं अंभः किं आसीत् ?:- Here the word Aṃbha (अंभ) refers to the Primeval water. The question arises is, why call this Primeval Water to be 'Gahana' (गहन) and 'Gaṃbhīra' (गंभीर)? Here 'Gaṃbhīra' (गंभीर) means unfathomably deep and 'Gahana' (गहन) means way too difficult to enter in. These adjectives indicate the non-differentiable nature of this Aṃbha (अंभ). That is why it is 'Primeval water'. The water which plays important role in the manifestation of the universe is essentially differentiable water and that same is described in next few verses. The water mentioned in this very first verse is different. It is not-disintegrable, uniquely unit and hence unfathomably deep water! Here, a doubt may arise: Taittirīya Brāhmaṇa (तैत्तिरीय ब्राह्मण ग्रंथ) gives 'आपो वा इदमग्रे सलिलमासीत्' as Śruti Vacana (श्रुति वचन). But this is not conflicting with the description given in first verse of Nāsadīya Sūkta (नासदीय सूक्त). This gets resolved due to 'Tadānīṃ' (तदानीं). Nāsadīya Sūkta (नासदीय सूक्त) provides account of pre-primordial time and what is given in the Śruti (श्रुति) is the beginning of the physical world. In fact, Nāsadīya Sūkta (नासदीय सूक्त) also discusses 'Apraketaṃ Salilaṃ' (अप्रकेतं सलिलं) in the next verse!

Thus Verse-1 of Nāsadīya Sūkta (नासदीय सूक्त) can be summarized as follows:

तदानीं,	At Pre-primordial times;
न असत् आसीत्,	There was no not-being;
नो सत् आसीत्,	Neither was there being and the conflicting pair, thus no Pārthiva (पार्थिव) world;
नो रजः,	There was no Antarikṣa (अन्तरिक्ष) world;
यत् परः विऽओम नो,	There was no Dyuloka (द्युलोक) world;
किं आवरीवः? कुह? कस्य शर्मन्?,	There was no cover of any kind and no force to bind the cover.
गहनं गभीरं अंभः किं आसीत्?,	The unfathomably deep 'Primeval water' also did not exist.

Drawing correspondences: Verse 1

It is important to understand the science hidden beneath and to draw correspondences of that with the modern scientific works. Well, this verse-1 describes the times which are Pre-primordial! Nothing existed which has some form and which belongs to this physical world. What exists eternally is beyond any description and hence in lieu of stating what existed, it is intelligently stated what did not exist. The description is extremely scientific though. It connects very well with the modern day physic and technology.

Out of the three worlds prescribed by Yāska (यास्क), the one where all the human beings reside, is the Pārthiva (पार्थिव) or the physical world or the Bhūloka (भूलोक). It has been captured in the most scientific manner in this verse.

R This physical world or 'Jagat' (जगत्) emerges out of 'Dvandva' (द्वंद्व = conflicting pair = duel) like 'Sat-Asat' (सत्-असत्=being-nonbeing). This is because Jagat (जगत्) is a moving thing. Motion arises out of Dvandva (द्वंद्व) and Dvandva (द्वंद्व) emerges out of motion. Thus, Dvandva (द्वंद्व) or duel and motion are interdependent. Therefore, Jagat (जगत्) is 'Relative' (सापेक्ष) and in fact, 'Relativity' (सापेक्षता) is the most fundamental norm of motion. This is in line with the Theory of Relativity proposed by Albert Einstein [67]. The theory of relativity proposed by Einstein has two components: Special Relativity and General Relativity. General relativity explains the law of gravitation and its relationship with the other forces that exist in Nature. The same is mentioned through the concept of आवरीवः in the first verse (मंत्र) of the Nāsadīya Sūkta (नासदीय सूक्त). Special relativity applies to physical phenomenon at particle level and explains the interactions at macro and micro level, but does not cover gravity. This special relativity is captured in the second verse (मंत्र) of the Nāsadīya Sūkta (नासदीय सूक्त).

Before addressing 'Relativity', let's examine the more fundamental things first. Please refer to Figure (3.4). The most fundamental form of duel (मूळ वा मूल द्वन्द्व) is captured in the first verse (मन्त्र) of Nāsadīya Sūkta (नासदीय सूक्त). In Nāsadīya Sūkta (नासदीय सूक्त) it is stated in the form of 'Sat-Asat' (सत्-असत् = being-not being). Through the mention of 'Rajas' (रजस्), Nāsadīya Sūkta (नासदीय सूक्त) verse 1 also captures the duel of 'Sattva-Tama' (सत्त्व-तम). There are two schools of thoughts from the 'Ṣaṭ Darśanas' (षट् दर्शने), namely Sāṅkhya (सांख्य) and Vedānta or Advaita (वेदान्त वा अद्वैत), which are predominant. We will consider these two schools of thoughts primarily in this thesis. The Sāṅkhya (सांख्य) philosophy gives 'Prakṛti-Puruṣa-duel-dual' (प्रकृति-पुरूष-द्वन्द्व-द्वैत) and Vedānta or Advaita (वेदान्त वा अद्वैत) philosophy give 'Śapala Brahma-Māyā-duel-dual' (शपल ब्रह्म-माया-द्वन्द्व-द्वैत) as the most fundamental form of duel or dual (द्वन्द्व वा द्वैत). The remaining expansion is as follows:

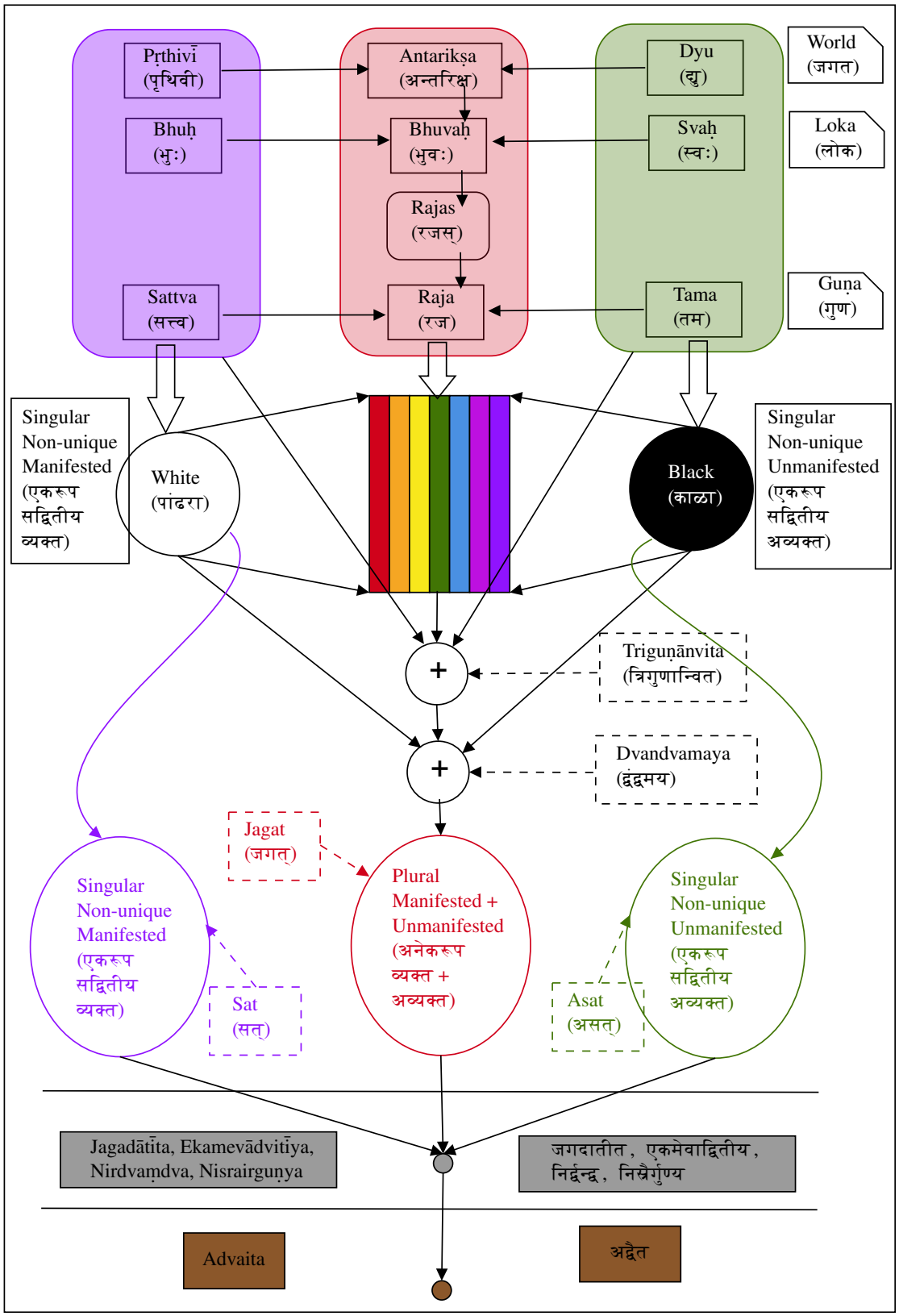


Figure 3.3: The use of 'Rajas' and the big picture

- The most fundamental form of duel/dual (द्वन्द्व वा द्वैत) is captured in the first verse (मन्त्र) of Nāsadīya Sūkta (नासदीय सूक्त)
- These most fundamental form of duel or dual (मूळ वा मूल - द्वन्द्व वा द्वैत) pairs are as follows:
  1. Mūla Dvandva (मूल द्वन्द्व): 'Sat-Asat-duel' (सत् - असत् = being-not being)from Nāsadīya Sūkta (नासदीय सूक्त) verse 1
  2. Mūla Dvandva (मूल द्वन्द्व): 'Sattva-Tama-duel' (सत्त्व - तम)from Nāsadīya Sūkta (नासदीय सूक्त) verse 1
  3. Mūla Dvaita (मूल द्वैत): 'Prakṛti-Puruṣa-dual' (प्रकृति - पुरुष - द्वन्द्व)from Sāṅkhya (सांख्य) philosophy
  4. Mūla Dvaita (मूल द्वैत): 'Śapala Brahma-Māyā-dual' (शपल ब्रह्म - माया - द्वैत)from Vedānta or Advaita (वेदान्त वा अद्वैत) philosophy

R

The pair is Dvandva (द्वन्द्व) pair when the entities are opposites and contradictory, like 'Sat' (सत्) and 'Asat' (असत्). One can not exist when the other exists and vice versa. When one arises, it annihilates the other.

The pair is Dvaita (द्वैत) pair when the entities are cooperating and complimentary, like 'Prakṛti' (प्रकृति) and 'Puruṣa' (पुरुष). They not only co-exist, but they also complement each other and in fact are incomplete without each other, thus forming a good pair.

- Dvandva (द्वन्द्व) and Motion (गति) are interoperable and interdependent
- This Dvandva (द्वन्द्व) manifests the non-moving 'Ajagat' (अजगत्) into moving 'Jagat' (जगत्)
- If the motion (गति) in the Jagat (जगत्) is considered as cause, it has three-fold effect as follows:
  1. Radiance (तप - तेज): This results into creation of Ākāśa (Aether or आकाश)
  2. Attraction-Repulsion-Duel (आकर्षणोत्सारण - द्वन्द्व): This fruitions into creation of Dik (Space or दिक्)
  3. Beginning-End-duel (जन्म - मृत्यू - द्वन्द्व): This effectuates into creation of Kāla (time or काल वा काळ)
- This is why Aether is termed as Ākāśa (आकाश), meaning समंतात् काश or glittering from all directions, as it originates from radiations or (तप - तेज)
- The connect between Ākāśa (Aether or आकाश) and Attraction-Repulsion-Duel (आकर्षणोत्सारण - द्वन्द्व) is important in formation of Dik (Space or दिक्)
- The connect between Dik (Space or दिक्) and Beginning-End-duel (जन्म - मृत्यू - द्वन्द्व) is critical in creation of Kāla (time or काल वा काळ)
- From Ākāśa (Aether or आकाश), the remaining four of the five Mahā-Bhūtas (पञ्च - महा - भूत) get created. This will be explained subsequently

All of this is captured comprehensively in Figure (3.4). Thus, everything begins with Dvandva/Dvaita (द्वन्द्व वा द्वैत). This is why even Lord Kṛṣṇa (कृष्ण) in Bhagavadgītā (भगवद्गीता) proclaims: 'प्रकृतिं पुरुषं चैव विध्ध्यनादी उभावपि'. From the modern physics the basic duel relationship between Electron-Proton is just like 'Prakṛti-Puruṣa-dual' (प्रकृति - पुरुष - द्वैत).

Now, lets connect these concepts with classical mechanics. Before doing that however, we must put few important facts in light. These facts clearly show how Bhāratīya Vedika (भारतीय वेदिक) thinking was divine, perfect, matured and way ahead of the counter western parts.

- Yāska clearly states that Āditya (the Sun) is the prime deity (प्रधान देवता). Thus, Vedic Ṛṣi knew that the universe was heliocentric and not geocentric.

- It was Polish astronomer Nicolaus Copernicus, who in year 1543 proved the universe being heliocentric. Till then, western world was under the impression of the world being geocentric! Those, like Galileo Galilei, who supported him were burnt to death!
- The the first verse (मन्त्र) of Nāsadīya Sūkta (नासदीय सूक्त) captures gravity through आवरीवः
- It was not before year 1686, when Sir Isaac Newton gave his profound theory on law of universal gravitation, the western world got familiarized with it.
- The theory relativity is also profoundly captured in Nāsadīya Sūkta (नासदीय सूक्त), which was given by Albert Einstein to the western world!

The moment we start discussion on the moving Jagat (गतिमान जगत), and particularly movement in it, we need to check the correlation of it with the modern mechanics. In Newtonian mechanics, the momentum is given as:

$$p = mv \tag{3.28}$$

where, p is the momentum, m is the mass and v is the velocity of the object. The famous Newton's second law of mechanics is stated as: "the rate of change of momentum of a body is directly proportional to the force applied, and this change in momentum takes place in the direction of the applied force". Thus,

$$F = \frac{dp}{dt} = \frac{d(mv)}{dt} \tag{3.29}$$

This gets further simplified from Equation (3.28) and (3.29) as:

$$F = m\frac{dv}{dt} = ma \tag{3.30}$$

For a given object with its position x, the first order derivative gives the velocity of the object.

$$v = \frac{dx}{dt} \tag{3.31}$$

Now, the second order derivative of x with respect to time t, it gives us acceleration a!

$$a = \frac{d^2x}{dt^2} \tag{3.32}$$

It is to be noted that using first order derivative we only generate velocity, which with mass can produce momentum. Thus, for generation of momentum, duel (द्वन्द्व) is absolute necessity. Although momentum is mass times velocity, acceleration is rate of change of velocity and this is captured by Teja or Tapa (तेज वा तप). This acceleration times mass then produces various Forces in moving Jagat (गतिमान जगत), with the likes of forces of attraction and repulsion (आकर्षणोत्सारण) for example. Thus, the model in the first verse of Nāsadīya Sūkta presenting the ladder of 'Dvandva-Gati-Teja-Ākāśa' (द्वन्द्व - गति - तेज - आकाश = duel-momentum-Force-Ether) fits in with the modern Newtonian Mechanics Model and particularly the second law of motion and law of conservation of moment.

R Two more brilliant pieces from Einstein can be wonderfully uncovered here.

1. In 1917, in his famous paper ‘Cosmological Considerations in the General Theory of Relativity’, Einstein gives his preferred cosmology as a model which is temporally infinite but spatially finite model! The spatial finite nature is captured by the ‘Sattva-Tama’ (सत्त्व - तम) duel (द्वन्द्व) and temporal infinite is captured by multi-fold ‘Rajas’ (रजस्)!
2. In his famous Theory of Relativity [[67]] convincingly showed that ‘time’ and ‘space’ are not absolute and independent of each other. If fact, they closely relate to each other and the same concept is brought out in Figure (3.4). Thus, Nāsadīya Sūkta emphatically and with great clarity brings out one of the most profound theories presented in modern physics in the last century!

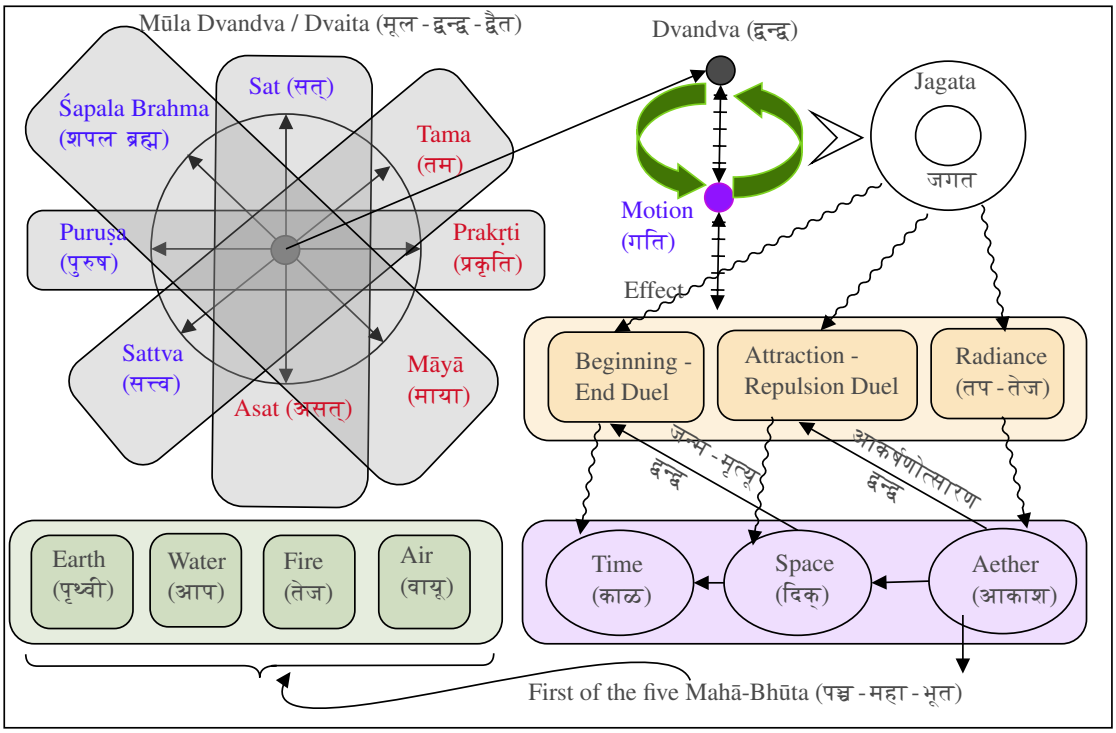


Figure 3.4: Jagat from the most Fundamental Duel/Dual or Dvandva/Dvaita

We know today, that Newtonian physics does not answer many questions and therefore there is need to go beyond. The next verse takes us in the realms which are beyond the classical Newtonian physics and mechanics.

3.3.2 Nāsadīya Sūkta - Verse Number 2

The second verse along with the anvaya (अन्वय) is as follows:

.. Ṛk No 2 ..

.. ऋक् क्र. २ ..

- न मृत्युरासीदमृतं न तर्हि न रात्र्या अह्न आसीत्प्रकेतः ।
  आनीदवातं स्वधया तदेकं तस्माद्धान्यन्न परः किं चनास ॥ १०.१२९.०२
- अन्वय - न । मृत्युः । आसीत् । अमृतं । न । तर्हि । न । रात्र्याः । अह्नः । आसीत् । प्रऽकेतः । आनीत् । अवातं । स्वधया । तत् । एकं । तस्मात् । ह । अन्यत् । न । परः । किं । चन । आस ॥ १०-१२९-०२
- शब्दार्थ - न = no । मृत्युः = Mortality । आसीत् = existed । अमृतं । न = Nor Immortality existed । तर्हि = At Pre-primordial times । न । रात्र्याः । अह्नः । आसीत् = Neither night nor day existed । प्रऽकेतः = Symbol giving distinction (between night and day) । तत् । एकं = That one and only one । स्वधया = by the inherent self-power । अवातं = windless or unpurturbed । आनीत् = was breathing । तस्मात् = Apart from that (one) । ह = Indeed । अन्यत् । परः = Another one । किं । चन । न । आस ॥ = did not exist १०-१२९-०२

Following Table (3.5) provides the views and interpretations of these 15 scholars on verse 2 of the present hymn.

Analytics 3.3.2 — Proposed decoding of Verse 2. We will now look at every word and try and decipher it towards one unified interpretation. The attempt is to blend the spiritual view with the scientific view and show that they have huge overlap and commonalities.

तर्हि**:-** This word is equivalent to the word तदानीं from the verse 1. This word is yet again indicator of the Pre-primordial time. Thus, we can conclude that the first two verses of hymn Nāsadīya Sūkta describe the situation at the time when this physical world (Jagat = जगत्) did not exist.

न मृत्युः आसीत् न अमृतं**:-** This indicates that neither mortality existed not immortality existed.

- It is important to note that Mṛtyu and Amṛta (मृत्यु आणि अमृत) yet again form Dvandva (द्वन्द्व) pair.
- In accordance with the meaning given by Sāyaṇācārya (सायणाचार्य) in Taittirīya Bhāṣya (तैत्तिरीय भाष्य) the meaning is taken in generic sense. Therefore generic Mortality-Immortality pair makes more sense than specific Life-Death.
- Here Mṛtyu and Amṛta (मृत्यु आणि अमृत) are equivalent to Asat and Sat (असत् आणि सत्) correspondingly.
- The theory of forming transcendence proposed by Ahitāgni Rājavāde however, it not acceptable to authors and hence we refute the same. (For example complementarity between Asat and Mṛtyu is flawed).
- Here Mṛtyu and Amṛta (मृत्यु आणि अमृत) also represent organic (सेंद्रिय) and inorganic (निरिंद्रिय) parts of the world. Still the question remains why this is mentioned in this verse? It is important to understand the significance here.
- In order to unlock this question, let's visit hymn no (10.72) of Ṛgveda (ऋग्वेदातील ऋचा क्र. १०.७२). This hymn is called either as 'Aditi Ātmakathana' (अदिति आत्मकथन) or 'Sṛṣṭyutpattisūkta' (सृष्ट्युत्पत्तिसूक्त). As the name suggest it is a tale of creation of universe (सृष्ट्युत्पत्ति) told by Aditi (अदिति).

Comparative Interpretations of scholars on Verse 2 of Nasadiya Sukta	
Name of the interpreter	Interpretation
Vivekananda [27]	Death was not then, nor immortality,The night was neither separate from day,But motionless did That vibrate Alone, with Its own glory one-Beyond That nothing did exist.
Max Mueller [28]	There was not death - hence was there naught immortal, There was no confine betwixt day and night; The only One breathed breathless in itself, Other than it there nothing since has been.
A. A. Macdonell [29]	Death then existed not nor life immortal; Of neither night nor day was any token. By its inherent force the One breathed windless: No other thing than that beyond existed.
Raimundo Panikkar [30]	There was no death then, nor yet deathlessness; of night or day there was not any sign. The One breathed without breath by its own impulse. Other than that was nothing at all.
Ralph Griffith [31]	Death was not then, nor was there aught immortal : no sign was there, the day's and night's divider. That One Thing, breathless, breathed by its own nature : apart from it was nothing whatsoever.
Jamison & Brereton [32]	Death did not exist nor deathlessness then. There existed no sign of night nor of day. That One breathed without wind through its inherent force. There existed nothing else beyond that.
Geldner [33]	Weder Tod noch Unsterblichkeit war damals; nicht gab es ein Anzeichen von Tag und Nacht. Es atmete nach seinem Eigengesetz ohne Windzug dieses Eine. Irgend ein Anderes als dieses war weiter nicht vorhanden.
A. L. Basham [34]	Then there were neither death nor immortality, nor was there then the torch of night and day. The One breathed windlessly and self-sustaining. There was that One then, and there was no other.
Wilson, HH: [35]	Death was not nor at that period immortality, there was no indication of day or night; THAT ONE unbreathed upon breathed of his own strength, other than THAT there was nothing else whatever.
Wendy Doniger O'Flaherty [36]	There was neither death nor immortality then. There was no distinguishing sign of night nor of day. That one breathed, windless, by its own impulse. Other than that there was nothing beyond.
Pandit RamGovind Trivedi [37]	उस समय मृत्यु नही थी, अमरता भी नही थी, रात और दिन का भेद भी नही था। वायु-शून्य और आत्मावलम्बन से श्वास-प्रश्वास-युक्त केवल एक ब्रह्म थे। उनके अतिरिक्त और कुछ नही था।
Siddheshwar Shastri Chitrav [38]	त्याकाळी मृत्यु (मृत्यूग्रस्त नाशिवंत सृष्टी) नसल्याने अमृतहि (अविनाशी पदार्थ) अस्तित्वात नव्हते. अहोरात्रिविभेदक काही साधनहि नव्हते. तत्कालीन अबोध सृष्टी स्वबलाने वायूशिवायच श्वासोच्छ्वास करीत स्फुरत होती. त्याखेरीज अथवा त्यापलीकदे अन्य काहीच नव्हते.
Krishnananda [39]	Then there was neither death nor immortality nor was there then the torch of night and day. The One breathed windlessly and self-sustaining. There was that One then, and there was no other.
Hari Damodar Velankar [40]	उस समय न मृत्यु था; न अमृतत्व भी। उसी प्रकार रात्रि या दिनका भी कोई चिन्ह नही था। उस समय (परमात्म तत्त्व) अकेला ही वातका अस्तित्व न होने पर भी स्वेच्छासे श्वासोच्छ्वास कर रहा था। सचमुच उससे दूसरी कोई वस्तु उस समय थी ही नही।
Sāyaṇācārya [41]	ननूक्तस्य प्रतिसंहारस्य संहर्त्रपेक्षत्वात् स एव .....

Table 3.5: Contemporary Interpretations: Verse 2 of Nāsadīya Sūkta

- Let's looks at few select verses (मन्त्र) from the hymn no (10.72) of Ṛgveda (ऋग्वेदातील ऋचा क्र. १०.७२):
  - देवानां युगे प्रथमे असतः सत् अजायत ।
    तत् आशाः अनु अजायन्त तत् उत्तानपदः परि ॥ ऋग्वेद १०.७२.३
  - भूः जज्ञे उत्तानपदः भुवः आशाः अजायन्त ।
    अदितेः दक्षः अजायत दक्षात् उ अदितिः परि ॥ ऋग्वेद १०.७२.४
  - यत् देवाः अदः सलिले सु संरब्धाः अतिष्ठत ।
    अत्र वः नृत्यतां इव तीव्रः रेणुः अपायत ॥ ऋग्वेद १०.७२.६
  - यत् देवाः यतयः यथा भुवनानि अपिन्वत ।
    अत्र समुद्रे आ गूळ्हं सूर्यं अजभर्तन ॥ ऋग्वेद १०.७२.७
  - अष्टौ पुत्रासः अदितेः ये जाताः तन्वः परि ।
    देवन् उप प्रैत सप्तभिः परा मा ॥ ऋग्वेद १०.७२.८
- The big picture is captured in Figure (3.5). It eventuates the sequence of creation of the universe.
- Initially (this is expressed by देवानां प्रथमे युगे in verse 10.72.3) the universe was not manifested (Asat = असत्).
- Form 'Asat' the manifested form of the universe, that is (Sat = सत्) got created (असतः सत् अजायत ॥).
- Form Sat = सत् all the three worlds came in existence namely Dyu (द्यु), Antarikṣa (अन्तरिक्ष) and Pārthiva (पार्थिव).
- From the Water (Adaḥ Salile = अदः सलिले from 10.72.6) emerged the eight sons of Aditi (Aṣṭau Putrāsaḥ Aditeḥ - अष्टौ पुत्रासः अदितेः from 10.72.8). These are:
  1. Sūrya (सूर्य)
  2. Bhaga (भग)
  3. Mārtāṇḍa (मार्ताण्ड)
  4. Vivasvāna (विवस्वान)
  5. Mitra (मित्र)
  6. Aryamā (अर्यमा)
  7. Savitā (सविता)
  8. Anśu (अन्शु)
- Out of these eight, seven went to Devāḥ (देवाः) as per 10.72.8.
- Aditi establishes Mārtāṇḍa (मार्ताण्ड) in Antarikṣa (अन्तरिक्ष).
- It is important to notice the similarity between Mārtāṇḍa (मार्ताण्ड) and Mṛtyu (मृत्यु)
- Mārtāṇḍa (मार्ताण्ड) creates time (t) or Kāla (काल). The moment time is created it embarks upon the concept of beginning and end. This is because what has start, it has to have end. This is the creation of Dvandva (द्वन्द्व) pair Mṛtyu and Amṛta (मृत्यु आणि अमृत).
- Mārtāṇḍa (मार्ताण्ड) also makes the earth rotate around it and this creates Dika (दिक् = space).
- This is in line with the concept of space-time put forth by Einstein.
- Mārtāṇḍa (मार्ताण्ड) imparts gravitational force that binds the Jagat (जगत्).
- From Vivasvāna (विवस्वान), Vaivasvata Manū (वैवस्वत मनू) and subsequently other human beings (मनुष्य प्रजा) gets created.
- From Vivasvāna (विवस्वान) also Uttānapada (अश्वत्थवृक्ष) is generated. This has its roots in the Antarikṣa (अन्तरिक्ष) or upward direction and its leaves and fruits in downward

direction, that is Pārthiva (पार्थिव). This nomenclature of Uttānapada (अश्वत्थवृक्ष) from verse 10.72.4, is similar to '(ऊर्ध्वमूल अधःशाखम्)' from (15.1) of Bhagavadgītā (भगवद्गीता) or Aśvatthavṛkṣa (अश्वत्थवृक्ष) from (1.14.7) of Ṛgveda.

- Form this Aśvatthavṛkṣa (अश्वत्थवृक्ष) all the plants and other beings get generated.
- The sequence is as follows: Unmanifested form (Asat = असत्) - Manifested form (Sat = सत्) - Dyu (द्यु) - Antarikṣa (अन्तरिक्ष) - Pārthiva (पार्थिव) - Water (Salila = सलिल) - eight sons of Aditi (Aṣṭau Putrāsaḥ Aditeḥ अष्टौ पुत्रासः अदितेः) - (1)Sūrya (सूर्य) - (2)Bhaga (भग) - (3)Mārtāṇḍa (मार्ताण्ड) - (4)Vivasvāna (विवस्वान) - (5)Mitra (मित्र) - (6)Aryamā (अर्यमा) - (7)Savitā (सविता) - (8)Anśu (अन्शु) - seven went to Devāḥ (देवाः) - Mārtāṇḍa (मार्ताण्ड) in Antarikṣa (अन्तरिक्ष) - Creation of time (t) or Kāla (काल) - Creation of Dik (दिक् = space) - Creation of gravitional force - Creation of Vaivasvata Manū (वैवस्वत मनू) and subsequently other human beings (मनुष्य प्रजा) - Creation of Aśvatthavṛkṣa (अश्वत्थवृक्ष) and all the plants and other beings.

न रात्र्याः अह्नः प्रकेतः आसीत्:- This description fits in with the earlier interpretation. The day (अह्नः) and night (रात्री) again form a duel pair (द्वन्द्व). The distinction of night from day occurs because of Mārtāṇḍa (मार्ताण्ड) in Antarikṣa (अन्तरिक्ष) which makes the earth revolve around it. This symbol (प्रकेतः) of distinction did not exist as Mārtāṇḍa (मार्ताण्ड) also did not exist in Pre-primordial times. The lack of distinction between day and night also indicates that there was no time-keeping (कालगणना), since time itself did not exist.

तत एकं:- After telling good account of what did not exist before, this is for the first time now the author of the hymn (सूक्तकार ऋषी) describes what existed then! Well, it was that 'one' and only one! That ONE:

- In Pārthiva (पार्थिव) world it is beyond any **duel** pair (द्वन्द्व) and hence is 'Nirdvandva' (निर्द्वन्द्व).
- In Antarikṣa (अन्तरिक्ष) world it is beyond any **dual** pair (द्वैत) and hence is 'Advaita' (अद्वैत).
- In Dyu (द्यु) world it is beyond any **double** pair (द्वय) and hence is 'Advaya' (अद्वय).

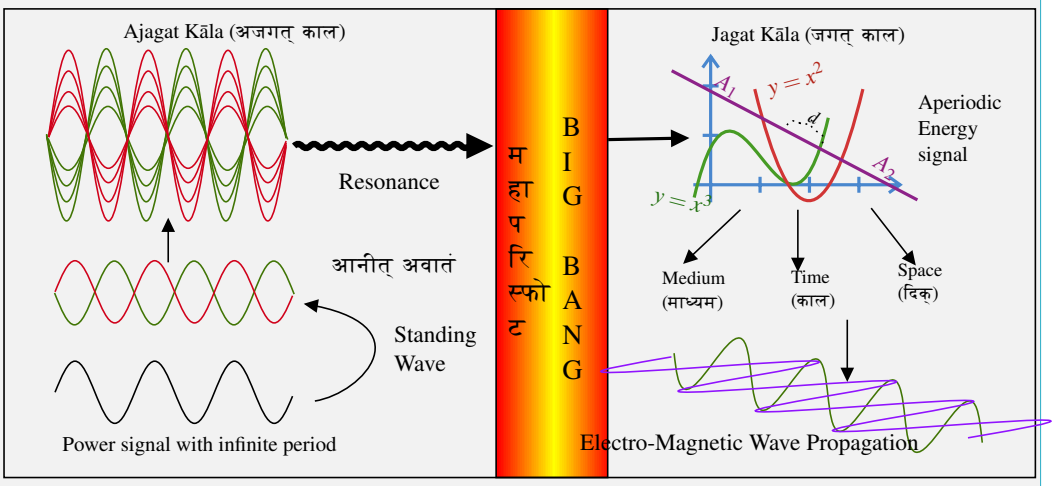
That ONE is also being indicated by four 'Mahā-Vākyas' (महा - वाक्ये)

1. 'Prajñānaṃ Brahma' (प्रज्ञानं ब्रह्म) from Aitareya Āraṇyaka (ऐतरेय आरण्यक) essentially describes that one unified ultimate Caitanya (चैतन्य)
2. 'Ahaṃ Brahma Asmi' (अहं ब्रह्म अस्मि) from Bṛhadāraṇyakopaniṣada (बृहदारण्यकोपनिषद) also decribes going beyond dualism by the word Asmi (अस्मि) and unison with that one unified ultimate Caitanya (चैतन्य)
3. 'Tat-Tvam-Asi' (तत् - त्वम् - असि) from Chāndogya (छान्दोग्य उपनिषद) also decribes union by the word Asmi (अस्मि) with that one unified ultimate Caitanya (चैतन्य)
4. 'Ayamātmā Brahma' (अयमात्मा ब्रह्म) from Māṇḍukya (माण्डुक्योपनिषद) again describes the same one unified ultimate Caitanya (चैतन्य)

स्वधया:- This is the most appropriate adjective to that one! That ONE is not dependent on anyone, therefore it is does not need anyone else to form a pair or either duel or dual. The existence of that ONE is through self-kernel and there is no external support system (आश्रयस्थान) that is needed. What does that ONE do with this self-power is discussed next.

आनीत् अवातं:- This is yet another adjective of that ONE that did exist at pre-primordial times. Apart from this ONE, no one else existed! But the present description of that ONE may sound contradictory or impossible. आनित means to breathe, but this breathing is extra-ordinary. Can we breath without oxygen? Of course not! But that ONE can! अवातं means

without वात or वायु (air) or प्राणवायु (air). That ONE breathes with self-power without any air. That ONE does not require any medium for breathing and that is why only that ONE existed then. Everything else in this Jagat (जगत्) is of duel/dual (द्वन्द्व वा द्वैत) form and is not comprehensively independent. Everything in the physical world is interdependent. Ether depends on the radiation and space-time depends on ether for example. This is why none of this existed at pre-primordial times. Here the question is how that ONE could breathe without any medium? This can be answered scientifically as follows:



Let's understand this from the signal processing perspective.

- In Ajagat Kāla (अजगत् काल) only that ONE existed, no one else
- This ONE is a periodic power signal with period = ∞
- Such signals have finite power (शक्ति) but infinite energy (ऊर्जा)
- They form standing waves which are stationary and can not transfer any power, thus they retain all the power.
- Standing waves being stationary, they do not propagate and does not require propagation medium.
- This explains आनीत् अवातं part of that ONE.
- In standing waves the troughs and crests, that is nodes and antinodes, they often collide and thus have excellent correlation.
- This correlation causes resonance. Since ∞ undergoes resonance, it results into 'Big-Bang' or (महा - परिस्फोट).
- The 'Big-Bang' or (महा - परिस्फोट) converts Power signal of ONE into Energy signal of many.
- Energy signals have finite energy, but zero average power.
- These waves require medium to propagate. For these energy waves time-space also gets created.
- In Jagat Kāla (जगत् काल) thus, all the transformations are governed by energy propagation.
- For energy propagation, electro-magnetic waves are formed.
- In electro-magnetic waves, electric and magnetic fields - which are perpendicular or orthogonal to each other- are combined for energy transmission.

This is how आनीत् अवातं in context with the ONE gets explained.
तस्मात् अन्यत् परः:- Apart from that ONE with finite power and infinite energy
ह किंचन न आसः- Indeed, nothing existed

Thus Verse-2 of Nāsadīya Sūkta (नासदीय सूक्त) can be summarized as follows:

तर्हि,	At Pre-primordial times;
न मृत्युः आसीत्,	There was no mortality;
न अमृतं,	Neither was there immortality and the conflicting pair, thus no Mārtāṇḍa (मार्ताण्ड);
नो रात्र्याः अह्नः,	There was no day and night;
न प्रकेतः आसीत्,	There was no space-time to mark it;
तत् एकं,	There existed only that ONE with finite power and infinite energy;
स्वधया आनीत् अवातं,	That ONE with the self-kernel could breathe without air, through standing wave;
तस्मात अन्यत् परः किंचन न आस,	Apart from that ONE and only one;
ह किंचन न आस,	Indeed nothing else existed.

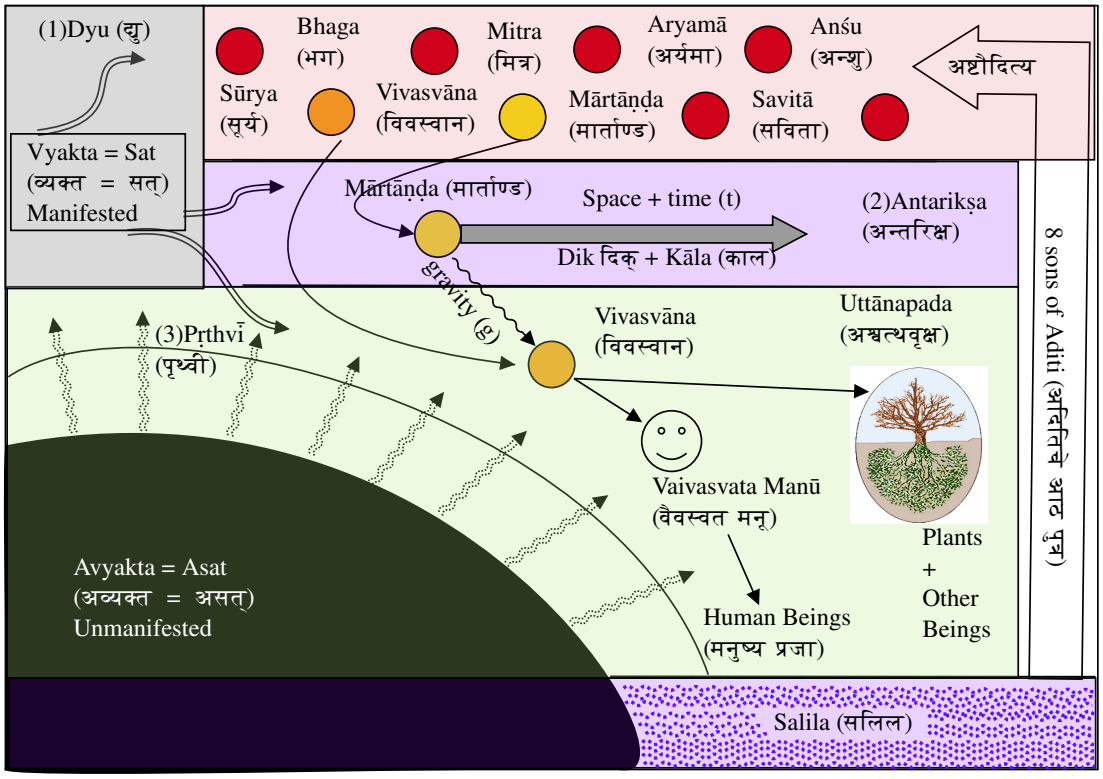


Figure 3.5: Creation of Universe as per Sṛṣṭyutpattisūkta (10.72) of Ṛgveda

Drawing correspondences: Verse 2

This verse presents one of the most fundamental principle based on which the other theories can be based. This is verse in which for the first time the Ṛṣi (ऋषि) has given clear account of what existed at pre-primordial time. What existed is beyond realms of any kind and hence is truly extra-ordinary. This is captured by स्वधया आनीत् अवातं.

- The concept that neither mortality existed nor immortality truly brings in the concept of Mārtāṇḍa (मार्ताण्ड).
- Because of Mārtāṇḍa (मार्ताण्ड), the day and night gets symbolized and marked, that also did not exist.
- Thus, in a very profound manner the enlightened seer has brought out the concept of time (t).
- The concept of space will be brought out in the next verse, however it is implied in time here.
- Now comes the most important part captured by स्वधया आनीत् अवातं.
- That ONE that did exist is truly extra-ordinary and the same is captured here.
- This Jagat (जगत्) or the physical world around us is full of energy transfers.
- In Pre-primordial times, before this physical world came into existence that ONE breathed without any air by virtue of its own special power.
- This special power is 'Periodic Power signal with period $t_p = \infty$'.
- This forms a unique standing wave for which even nodes and anti-nodes are at ∞ locations.
- Standing waves are the waves which do not propagate and hence are also termed as Stationary waves.
- Standing wave pattern is a vibrational wave pattern.
- It can not be just coincidental that Swāmī Vivekānanda has used word 'vibration' to describe this extra-ordinary phenomenon! [[27]].
- The remarkable things about this standing wave is, for any standing wave to occur interference between two waves in needed, here these two waves are embedded in that ONE and only one!
- While getting manifested one becomes Sat and other becomes Asat, for example.
- Standing waves do not transmit any energy
- The importance of this standing waves lies in the fact that while vibrating, the oscillations coincide with the frequencies and the nodes and antinodes are perfectly matched.
- This leads us to phenomenon of resonance!
- It is this resonance that eventuates into 'Big-bang' or (महा - स्फोट). This will be explained subsequently.
- Here, it is important to note that the seed for 'Big-bang' is sowed in this verse!

Calculations 3.1 — Bhāratīya Kālagaṇanā. Since we have discussed how Mārtāṇḍa (मार्ताण्ड) created Mṛtyū (मृत्यू) and effectively 'time(t)' (Kāla = काल), and that got manifested into 'Space' (Dik = दिक्), let's discuss Bhāratīya Kālagaṇanā (भारतीय कालगणना) here. Bhāratīya Kālagaṇanā (भारतीय कालगणना) is so profound that it also gives us good estimate of time since The-Big-Bang or महा - परि - स्फोट.

- In order to understand Bhāratīya Kālagaṇanā (भारतीय कालगणना), we need to put the units in place first.
- We refer to first Aṃśa (अंश) of Viṣṇu-Purāṇa (विष्णु - पुराण) to set the units in place.
- The 19^{th} Śloka (श्लोक) from chapter 3 (तृतीय अध्याय) of Viṣṇu-Purāṇa is as follows:
- अष्टौ शतसहस्राणि दिव्यया संख्यया स्मृतम् ।
  द्विपञ्चाशत्तथान्यानि सहस्राण्यधिकानि तु ॥ विष्णु - पुराण ३.१९

- This Śloka (श्लोक) clearly states that 1 Manvantara = 8,52,000 years. But these are 'Deva-Saṃvatsaras' (देव संवत्सर) or 'Deva-Varṣas' (देव वर्षे)
- Now, the 20^{th} and 21^{th} Śloka (श्लोक) from chapter 3 (तृतीय अध्याय) of Viṣṇu-Purāṇa is as follows:
- त्रिंशत्कोट्यस्तु सम्पूर्णाः संख्याताः संख्यया द्विज ।
  सप्तषष्टिस्तथान्यानि नियुतानि महामुने ॥ विष्णु - पुराण ३.२०
- विंशतिस्तु सहस्राणि कालोऽयमधिकं विना ।
  मन्वन्तरस्य सङ्ख्येयं मानुषैर्वत्सरैर्द्विज ॥ विष्णु - पुराण ३.२१
- From above two verses, 1 Manvantara = 30,67,20,000 years (30 crore, 67 lac, 20 thousand). These are human years!
- Thus, $(30,67,20,000 \div 8,52,000 = 360)$ is the conversion factor.
- Therefore, 1 Deva-Varṣa = 360 Sūrya-Varṣa
- Let's now put together Caturyuga (चतुर्युग) in terms of Deva-Varṣa (देव - वर्ष)

Yuga (युग),	Deva-Varṣa (देव - वर्ष)	Sūrya-Varṣa (सूर्य - वर्ष);
Satya/Kṛta (सत्य वा कृत),	4800	17,28,000;
Tretā (त्रेता),	3600	12,96,000;
Dvāpara (द्वापर),	2400	8,64,000;
Kali (कलि),	1200	4,32,000.

- Thus, 1 Caturyuga (चतुर्युग) = (4800+3600+2400+1200) = 12,000 Deva-Varṣa (देव - वर्ष)
- Therefore, 1 Caturyuga (चतुर्युग) = 12000 × 360 = 43,20,000 Sūrya-Varṣa (सूर्य - वर्ष)
- 12,000 divine years (Deva-Varṣa) = 4 Yugas = 1 Mahā-Yuga = 43,20,000 human years
- The 18^{th} Śloka (श्लोक) from chapter 3 (तृतीय अध्याय) of Viṣṇu-Purāṇa is as follows:
- चतुर्युगाणां संख्याता साधिका ह्येकसप्ततिः ।
  मन्वन्तरं मनोः कालः सुरादीनां च सत्तम ॥ विष्णु - पुराण ३.१९
- Now, as per 18^{th} Śloka (श्लोक) from chapter 3 (तृतीय अध्याय) of Viṣṇu-Purāṇa, 1 Manvantara (मन्वन्तर) is little more than 71 Caturyugas (चतुर्युगे). This little more is the Sandhikāla (सन्धिकाल) and is captured by term 'Sādhikā' (साधिका = स अधिका) in this 18^{th} Śloka.
- Now, 1 Caraṇa (चरण) = 1200 Deva-Varṣa (देव - वर्ष)
- Let's calculate Kalpa (कल्प) now.
- 1 Kalpa (कल्प) = 1000 Caturyugas (चतुर्युगे) = 1000 Mahā-Yugas (महा - युगे) as per Viṣṇu-Purāṇa (विष्णु - पुराण)
- 1 Kalpa (कल्प) includes 14 Manvantaras (मन्वन्तरे) + the closure of Sandhikāla (सन्धिकाल).
- 1 Manvantara (मन्वन्तर) includes 71 Caturyugas + the closure of Sandhikāla (सन्धिकाल).
- $\therefore$ 1 Kalpa (कल्प) = 14 × 71 = 994 Caturyugas (चतुर्युगे) + the closure of Sandhikāla (सन्धिकाल).
- This indicates the closure of Sandhikāla (सन्धिकाल) should be of (1000-994) = 6 Caturyugas (चतुर्युगे) = 6 Mahā-Yugas (महा - युगे) —— (a)
- Let's confirm this!
- As per Viṣṇu-Purāṇa (विष्णु - पुराण), 1 Sandhikāla (सन्धिकाल) is of the duration of Satya Yuga (सत्य युग) = 4 Caraṇas (चरण) = 4800 Deva-Varṣas (देव - वर्षे)
- This Sandhikāla (सन्धिकाल) is the transition phase and happens after every Manvantara (मन्वन्तर) and also before and after the first and last Manu (मनु).
- There are total 14 Manvantaras (मन्वन्तरे), and $\therefore$ 15 Sandhikaala (सन्धिकाल).
- Thus, TOTAL Sandhikāla (एकूण सन्धिकाल) = 15 × 4 Caraṇas (चरण).
- $\therefore$, TOTAL Sandhikāla (एकूण सन्धिकाल) = 15 × 4 × 1200 Deva-Varṣa (देव - वर्ष) = 72,000

Deva-Varṣa (देव - वर्ष)

- ∴ TOTAL Sandhikāla (एकूण सन्धिकाल) = (72,000/12,000) = 6 Caturyuga (चतुर्युग), ∵ 1 Caturyuga (चतुर्युग) = (4800+3600+2400+1200) = 12,000 Deva-Varṣa (देव - वर्ष)
- This matches with (a)!
- Thus, 1 Kalpa (कल्प) = 1000 Caturyugas (चतुर्युगे) = 4,320,000,000 solar years!

We will subsequently show that these calculations match with estimate by Plank (incorporating Hubble constant) and the rough estimate for the time since The-Big-Bang (महा - परि - स्फोट) is $\approx 13,799,000,000$ solar years!

■

Now, let's examine verse-3 of Nāsadīya Sūkta.

3.3.3 Nāsadīya Sūkta - Verse Number 3

The third verse along with the anvaya (अन्वय) is as follows:

.. Ṛk No. 3 ..

.. ऋक् क्र. ३ ..

- तमं आसीत्तमसा गूळ्हमग्रेऽप्रकेतं सलिलं सर्वमा इदम् ।
  तुच्छ्येनाभ्वपिहितं यदासीत्तपसस्तन्महिनाजायतैकम् ॥ १०.१२९.०३
- अन्वय - तमः । आसीत् । तमसा । गूळ्हं । अग्रे । अप्रऽकेतं । सलिलं । सर्वं । आः । इदं । तुच्छ्येन । आभु । अपिऽहितं । यत् । आसीत् । तपसः । तत् । महिना । अजायत । एकम् ॥ १० - १२९ - ०३
- शब्दार्थ - तमः = UnManifested । आसीत् = existed । तमसा = darkness, obscure । गूळ्हं = enigmatic, unfathomable, camouflaged । अग्रे = At primordial times, beginning of the creation of the universe । अप्रऽकेतं = Incomprehensible । सलिलं = moving entity, water । इदं । सर्वं = All this । आः = existed । तुच्छ्येन = empty or void । आभु = moving entity, water, immense power । अपिऽहितं = covered, concealed । यत् । तत् = this and that । आसीत् = existed । तपसः । महिना । अजायत = Born out of greatness of penance resulting in energy amplification and transformation । एकम् ॥ = Undifferentiable १० - १२९ - ०३

Following Table (3.6) provides the views and interpretations of these 15 scholars on verse 3 of the present hymn.

Analytics 3.3.3 — Proposed decoding of Verse 3. This according to us is the most difficult verse to decipher. We will now look at every word and try and decipher it towards one unified interpretation. The attempt is to blend the spiritual view with the scientific view and show that they have huge overlap and commonalities.

Here, it should be noted that the first two verses (मंत्र) of Nāsadīya Sūkta (नासदीय सूक्त) discuss the situation at Pre-primordial times. Now, verses 3-4-5 will discuss the Primordial time. This is

Comparative Interpretations of scholars on Verse 3 of Nasadiya Sukta	
Name of the interpreter	Interpretation
Vivekananda [27]	At first in darkness hidden darkness lay, Undistinguished as one mass of water, Then That which lay in void thus covered, A glory did put forth by Tapah!
Max Mueller [28]	Darkness there was, and all at first was veiled in gloom profound, - an ocean without light. - The germ that still lay covered in the husk burst forth, one nature, from the fervent heat.
A. A. Macdonell [29]	Darkness there was at first by darkness hidden; Without distinctive marks, this all was water. That which, becoming, by the void was covered, That One by force of heat came into being.
Raimundo Panikkar [30]	Darkness was there, all wrapped around by darkness, and all was Water indiscriminate, Then that which was hidden by Void, that One, emerging, stirring, through power of Ardor, came to be it;
Ralph Griffith [31]	Darkness there was : at first concealed in darkness this All was indiscriminated chaos. All that existed then was void and formless : by the great power of Warmth was born that Unit.
Jamison & Brereton [32]	Darkness existed, hidden by darkness, in the beginning. All this was a signless ocean. When the thing coming into being was concealed by emptiness, then was the One born by the power of heat.
Geldner [33]	Im Anfang war Finsternis in Finsternis versteckt;all dieses war unkenntliche Flut. Das Lebenskrige, das von der Leere eingeschlossen war, das Eine wurde durch die Macht seines heissen Dranges geboren.
A. L. Basham [34]	At first there was only darkness wrapped in darkness. All this was only unilluminated water. That One which came to be, enclosed in nothing, arose at last, born of the power of heat.
Wilson, HH: [35]	There was darkness covered by darkness in the beginning, all this (world) was undistinguishable water; that empty united (world) which was covered by a mere nothing, was produced through the power of austerity.
Wendy Doniger O'Flaherty [36]	Darkness was hidden by darkness in the beginning; with no distinguishing sign, all this was water. The life force that was covered with emptiness, that one arose through the power of heat.
Pandit RamGovind Trivedi [37]	सृष्टि के प्रथम अन्धकार (वा माया-रूपी अज्ञान) से अन्धकार (वा जगतकारण) ढका हुआ था। सभी अज्ञात और सब जलमय (वा अविभक्त) था। अविद्यमान वस्तु के द्वारा वह सर्वव्यापी आच्छन्न था। तपस्या के प्रभाव से वही एक तत्त्व उत्पन्न हुआ।
Siddheshwar Shastri Chitrav [38]	समस्त जल तमोव्याप्त आणि भेदाभेदविरहित असून तपमहिम्याने प्रकट झालेले सर्वव्यापी ब्रह्म (आभु) फोल मायेने (तुच्छ) आच्छादिले होते.
Krishnananda [39]	At first there was only darkness wrapped in darkness. All this was only unillumined water. That One which came to be, enclosed in nothing, arose at last, born of the power of heat.
Hari Damodar Velankar [40]	सृष्टिके पूर्व अन्धेरेने ही अन्धेरेको ढक लिया था और यह सारा विश्व जलमय होकर उसमे किसीका कोई चिन्ह प्रतीत नही होता था। वह प्रादुर्भूत होनेवाला (परमात्वतत्त्व) तुच्छ (जल) से (उस समय) आच्छादित हो गया था और अनन्तर तपकी महिमासे वही एक तत्त्व (जगद्रूप) मे प्रकट हो गया।
Sāyaṇācārya [41]	ननूक्तप्रकारेण यदि पूर्वमिदं जगन्नसीत् कथं तर्हि.....

Table 3.6: Contemporary Interpretations: Verse 3 of Nāsadīya Sūkta

the time when the universe comes into existence and the process and sequence of the universe getting manifested in mentioned now.

अग्रे:-
- This mention of time is different than the first two verses. The words 'तदानीं' and 'तर्हि' from the first two verses indicate the time which is Pre-primordial. When nothing existed except that ONE.
- This word 'अग्रे' indicates the primordial time. So this is the time of actual beginning of the universe.
- This is that time when everything is still unmanifested, however now that ONE is to become many!
- This is time just after the Big-Bang or महा - परिस्फोट.

तमः आसीत्:-
- Now this verse presents an account of how the manifestation happened and what existed at primordial times.
- Here word 'तमः' indicates lack of motion or गति.
- Thus, 'तमः' essentially indicates the unmanifested form or the 'Asat' (असत्).
- At primordial time what gets created first or what existed first was 'Asat'! This is in line with the earlier discussion, depicted in Figure (3.5).
- From Asat (असत्) what germinates is Sat (सत्).

> **R** A word of caution! It must be understood here, that Asat (असत्) only and strictly means 'Unmanifested' and Sat (सत्) only and strictly means 'Manifested'. These words should not be misunderstood with the meaning of these words in context with what gets used by Sāṅkhya and Advaita schools of philosophies to establish the well known doctine of 'Sat-Kārya-Vāda' (सत् - कार्य - वाद).

तमसा गूळ्हं इदं सर्वं अप्रकेतं सलिलं आः:-
- Here तमसा means Asat (असत्) or the darkness due to lack of any motion (गति).
- This Asat is again obscurely incomprehensible expressed by गूळ्हं.
- Now let's decipher सलिलं. This word is derived from root 'sal gatau = gatimaana ho.ne' or to become moving.
- Thus, सलिलं clearly depicts the Sat () or the moving physical world.
- This सलिलं however, is अप्रकेतं or non-differentiable as it gets camouflaged by the darkness of Asat.
- सलिलं is also 'Mahattatva' (महत्तत्व).
- The word pair इदं सर्वं means the physical world.
- आः is the Tṛtīya Puruṣa Ekavacana (तृतीय पुरुष एकवचन) of root 'Asa' (अस). It indicates existence.
- Since the most fundamental movement is captured in water, सलिलं also means water.

यत् तुच्छ्येन आभु अपिहितं आसीत् तत् एकं तपसः महिना अजायत:-
- Here word आभु indicated water and essentially symbolizes movement in the Jagat (जगत्).
- This आभु is the same अंभः from verse-1 that did not exist at pre-primordial time.
- This आभु was covered up (अपिहितं) by तुच्छ or void. This तुच्छ is equivalent to Māyā (माया) of Vedānta (वेदान्त) school of philosophy.
- एकं here is different than ONE that existed at pre-primordial times. This एकं is in primordial time and is also called as 'Śapala Brahma' (शपल ब्रह्म) in Vedānt and 'Puruṣa' (पुरुष) in Sāṅkhya philosophy. Whereas the ONE that existed at pre-primordial time is 'Turīya Brahma' (तुरीय ब्रह्म).

- This एकं performs Tapa (तप) or radiates exceptional haet power.
- Because of the greatness of this radiation heat, the manifested Jagat (जगत्) was born!

This is indeed in line with Figure (3.5), but this is also in tune with doctrine of 'Trivṛtkaraṇa' (त्रिवृत्करण सिद्धान्त) given in Chāndogyopaniṣada (छान्दोग्योपनिषद) or its extended version called 'Pañcīkaraṇa' (पञ्चीकरण) as per Pañcadaśī (पञ्चदशी).

Thus Verse-3 of Nāsadīya Sūkta (नासदीय सूक्त) can be summarized as follows:

अग्रे,	At Primordial time (beginning of universe);
तमः आसीत्,	There was Asat (Unmanifested world);
तमसा गुळ्हं इदं सर्वं,	All this was motionless, dark and hence was obscure and incomprehensible;
यत् तुच्छ्येन अपिहितं,	What illusionary void covered;
आभु आसीत तत्,	the moving water was this;
तत् एकं,	This one is with finite energy;
तपसः महिन अजायत,	That world was born out of, the greatness and depth of heat radiations.

Drawing correspondences: Verse 3

Let's do this with reference to Figure (3.6).

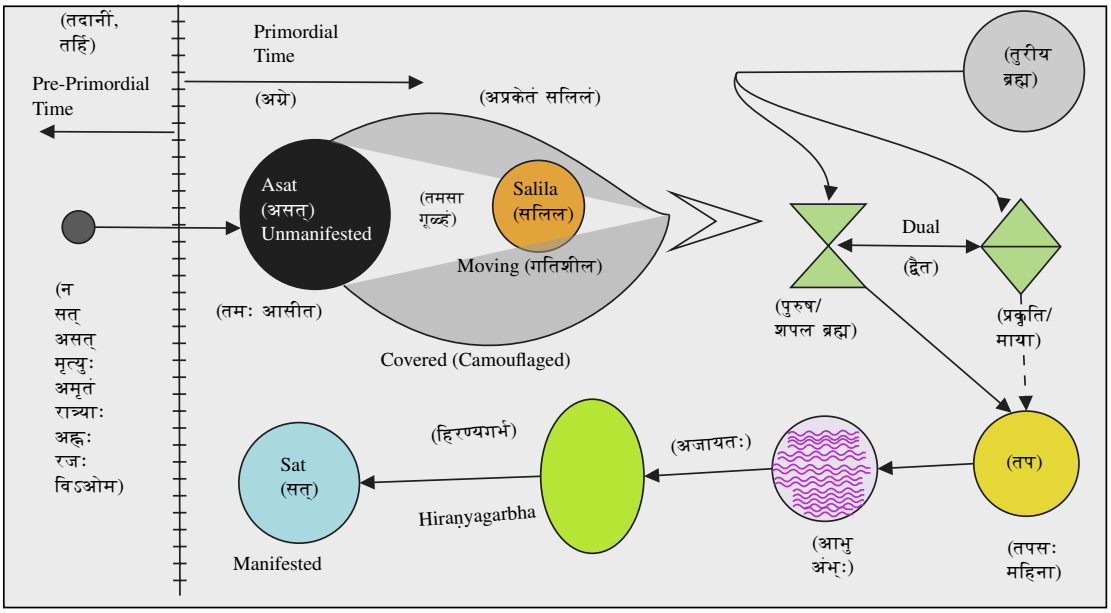


Figure 3.6: Sequence of Creation of Universe as per verse-3 of Nāsadīya Sūkta in Ṛgveda

There is good clarity on the fact that this verse-3 discusses the primordial time. The first two verses discussed the pre-primordial time when only ONE existed with its own kernel and it was a periodic power signal with period being ∞. The time described in this verse is the time when the creation of the universe actually begins. As per the sequence the first thing that comes into existence is Asat (असत्).

This is perfectly in compliance with vivid theories which emerge out of Vedika literature (वेदिक

वाङ्मय). For example, Taittirīyopaniṣad (तैत्तिरीयोपनिषद्) declares, 'असद्वा इदमग्र आसीत्ततो वै सदजायत', which means 'Initially Asat (असत्) existed and from that Sat (सत्) was produced'. We have already stated this, but we reproduce it here once again to reconfirm appropriate understanding of nomenclature.

R A word of caution! It must be understood here, that Asat (असत्) only and strictly means 'Unmanifested' and Sat (सत्) only and strictly means 'Manifested'. These words should not be misunderstood with the meaning of these words in context with what gets used by Sāṅkhya and Advaita schools of philosophies to establish the well known doctrine of 'Sat-Kārya-Vāda' (सत् - कार्य - वाद).

Thus, what we wish to state it, this sequence of creation does not refute 'Sat-Kārya-Vāda' (सत् - कार्य - वाद) and the context must be understood for clarity in comprehension.

Sat (सत्) and Asat (असत्) in the context of present Nāsadīya Sūkta only and essentially means manifested and unmanifested respectively. Sat (सत्) and Asat (असत्) in the context of six philosophical schools of ancient Bhārata (षट् दर्शने) means being and not-being respectively. In context with the cause-effect model (कार्य कारण भाव) the effect can originate only from the cause if the cause has that potential. For example, gold ornaments (effect) can be created from raw gold (cause with potential), but not from soil (cause with no potential). Therefore, the doctrine of 'Sat-Kārya-Vāda' (सत् - कार्य - वाद) suggests that this world (being 'Sat' =being), could have originated only from 'Sat' (potential) cause. With good clarity on this, we now move on to see close connection of this with evolutionary pattern with doctrine of 'Trivṛtkaraṇa' (त्रिवृत्करण सिद्धान्त) given in Chāndogyopaniṣad (छान्दोग्योपनिषद्) or its extended version called 'Pañcīkaraṇa' (पञ्चीकरण) as per Pañcadaśī (पञ्चदशी).

The concept of 'Trivṛtkaraṇa' is depicted in Figure (3.7).

- त्रिवृत्करणसिद्धान्त that is doctrine of 'Trivṛtkaraṇa' appears in Chāndogyopaniṣad (छान्दोग्योपनिषद्)
- A Ṛṣi (ऋषि) named Uddālaka Āruṇi (उद्दालक आरुणि) states this
- The doctrine proclaims the sṛṣṭī (सृष्टी) being created out of three fundamental entities (hence Trivṛt or त्रिवृत्)
- These are: 'Teja' (तेज), 'Jala' (जल) and 'Anna' (अन्न)
- Human beings (मनुष्य प्राणी) is made of 16 units which emerge out of the 3 fundamental, hence is called as 'Ṣoḍaśakalā' (षोडशकला).

The mathematical details of 'Trivṛtkaraṇa' is expressed in Figure (3.8).

'Trivṛtkaraṇa' (त्रिवृत्करण सिद्धान्त) as given in Chāndogyopaniṣada (छान्दोग्योपनिषद) gets extended and becomes 'Pañcīkaraṇa' (पञ्चीकरण). The complete process of 'Pañcīkaraṇa' (पञ्चीकरण) is given in detail in Pañcadaśī (पञ्चदशी) [42]. The process is shown in Figure (3.9), which is taken as is from [42] with slight modification by the author.

Thus, we conclude that the sequence given in verse 3 of Nāsadīya Sūkta is very much in line with the theories proposed in 'Trivṛtkaraṇa' (त्रिवृत्करण सिद्धान्त) from Chāndogyopaniṣada (छान्दोग्योपनिषद) and it's extension, 'Pañcīkaraṇa' (पञ्चीकरण) as per Pañcadaśī (पञ्चदशी).

It is indeed a matter of logical conclusion to see excellent agreement between 'Big-Bang-Theory' [1–3, 20–23] and Nāsadīya Sūkta. This is neatly depicted in Figure (3.10), the imaginary picture of The Big Bang used is taken from Wikipedia [19]. The sequence of formation namely Bang (स्फोट), Dark Ages (असत्), Big Bang Expansion (सत्).

Hiraṇyagarbha

- We have now formed consensus on the fact that from Asat (असत्), sat (सत्) was produced. Taittirīyopaniṣad (तैत्तिरीयोपनिषद्) in (2.7.1) proclaims: 'असद्वा इदमग्र आसीत्ततो वै

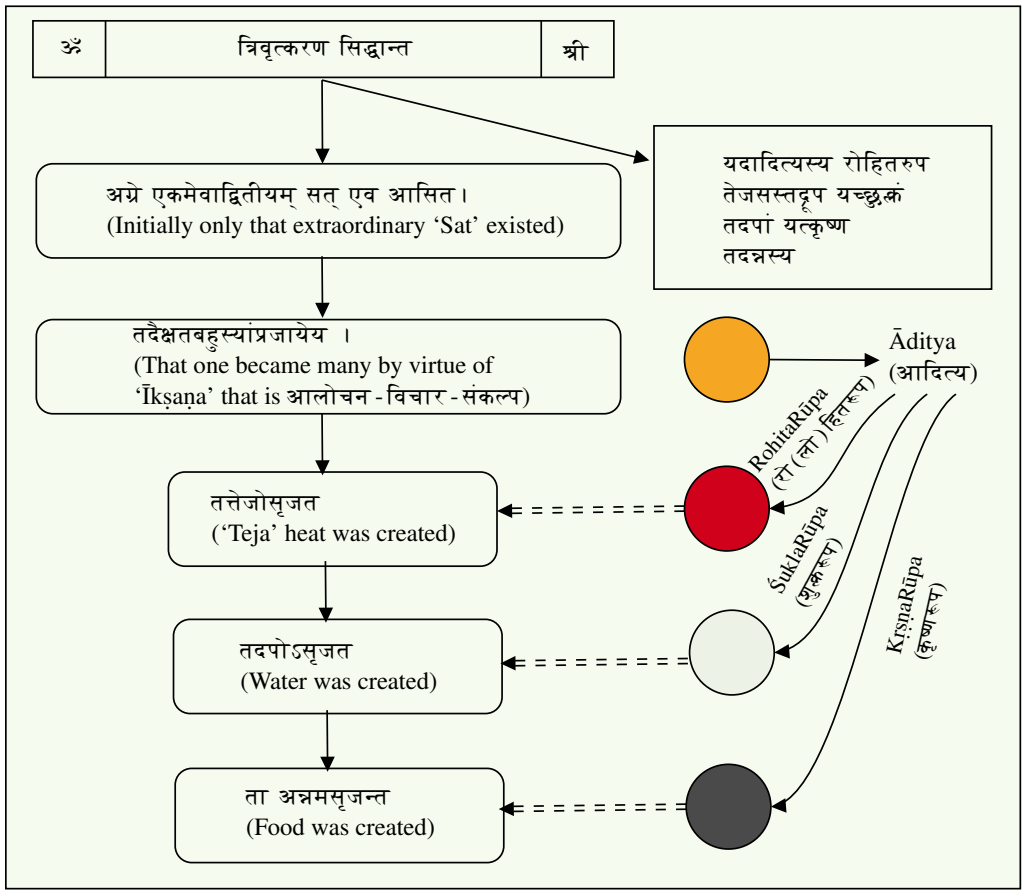


Figure 3.7: 'Trivṛtkaraṇa' as it appears in Chāndogyopaniṣada

सदजायत' and Bṛhadāraṇyakopaniṣad (बृहदारण्यकोपनिषद्) in (1.2.1) proclaims: 'नैवेह किंचनाग्र आसीन्मृत्युनैवेदमावृतमासीत्'. This is captured by 'तमसा गूळ्हं' in this verse.

- In this process the sequence is critically driven by Hiraṇyagarbha (हिरण्यगर्भ), hence let's explore it more.
- 'अप एव ससर्जादौ तासु वीर्यमवाकिरत्। तदंडमभवद्धैमं सूर्यकोटिसमप्रभम्॥' [1] From Teja (तेज), Apa (अप) got created and from Apa (अप) an egg shaped golden construct got created. This was Hiraṇyagarbha (हिरण्यगर्भ)!
- Hiraṇyagarbha (हिरण्यगर्भ) removed the darkness from the water and that created Ākāśa (आकाश).
- This Hiraṇyagarbha (हिरण्यगर्भ) is no differen than Brahmāṇḍa (ब्रह्माण्ड).
- The reason behind the creation of Brahmāṇḍa (ब्रह्माण्ड) can be analyzed in four different ways:
  1. Upādāna Kāraṇa (उपादान कारण=Root Cause): The Salila (सलिल = moving or गतिमान entity) that was born out of the 'Tapa' (तप) or the heat energy that emerged out of resonance from that ONE which existed at Pre-primordial times.
  2. Nimitta Kāraṇa (निमित्त कारण=Incidental Cause): 'Kāma' (काम=perturbation) as we will see in the next verse.

[1] मनुस्मृति. १.८

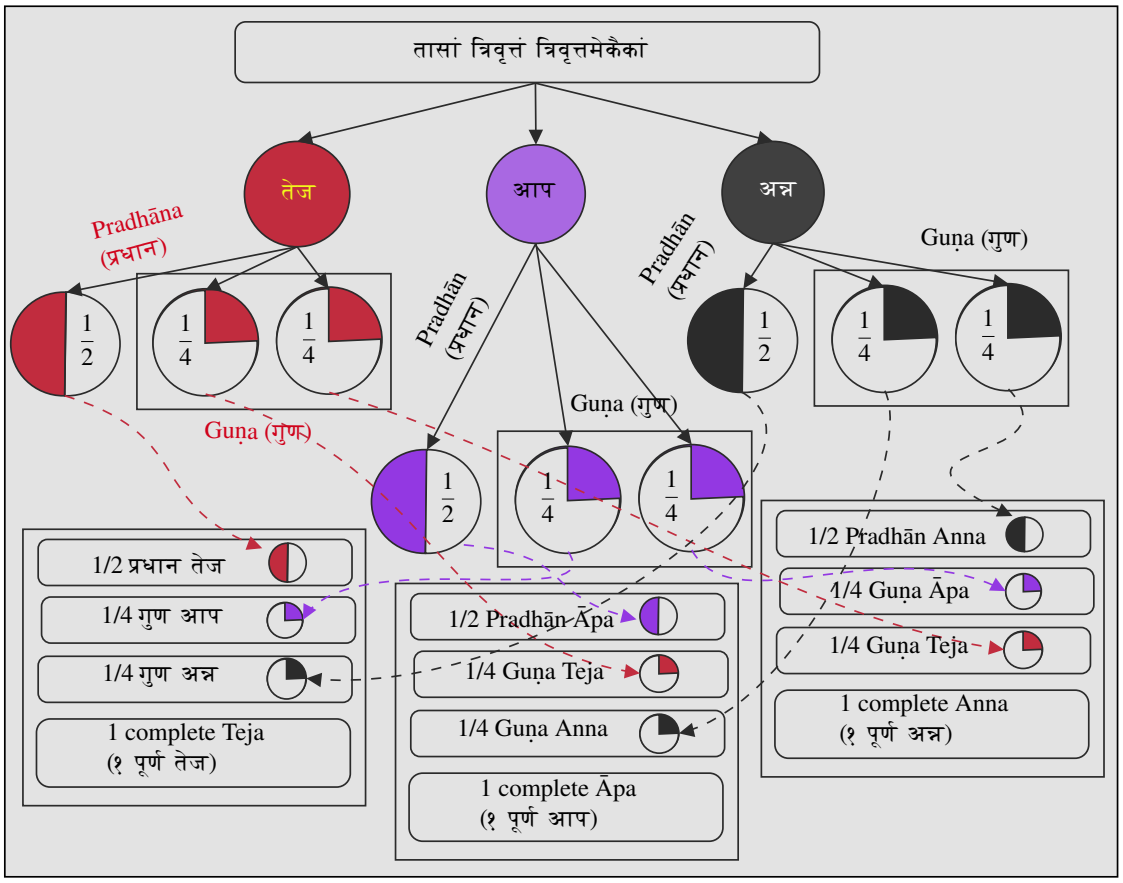


Figure 3.8: 'Trivṛtkaraṇa' Mathematics

3. Rūpa Kāraṇa (रूप कारण=Formal Cause): The very process of 'Trivṛtkaraṇa' (त्रिवृत्करण सिद्धान्त) as seen earlier.
4. Phala Kāraṇa (फल कारण=Consequential Cause): The creation of entire universe thus realizing बहु स्यां प्रजायेय.

- Taittirīya Śruti (तैत्तिरीय श्रुति) in (6.2.2.5) - 'प्रजापतिर्वै हिरण्यगर्भः' confirms that Hiraṇyagarbha (हिरण्यगर्भ) is Brahmadeva (ब्रह्मदेव) who created this universe.
- To explore more about Hiraṇyagarbha (हिरण्यगर्भ), there is a Sūkta (सूक्त) in Triṣṭubh Chanda (त्रिष्टुभ् छन्द) by Prajāpatiputra (प्रजापतिपुत्र) and Devatā (देवता) is 'Ka' '(ka)'! Let's explore this more.

Sūkta of 'Ka' Devatā

- In this hymn the word 'Kasmai' (कस्मै) is used with intended pun.
- 'कस्मै देवाय हविषा विधेम' is a question and also an answer!
- For question कोऽयं देवः ?, the answer is अयं देवः कः.
- Thus this 'Ka Devatā' (क देवता) is Prajāpati (प्रजापति) = Sūrya (सूर्य) = Tvaṣṭā (त्वष्टा) = Viśvakarmā (विश्वकर्मा) = Ātmā (आत्मा) = Hiraṇyagarbha (हिरण्यगर्भ) = Suparṇa (सुपर्ण) = Puruṣa (पुरुष).
- The first verse (मन्त्र वा ऋचा) in this hymn (सूक्त) is as follows:

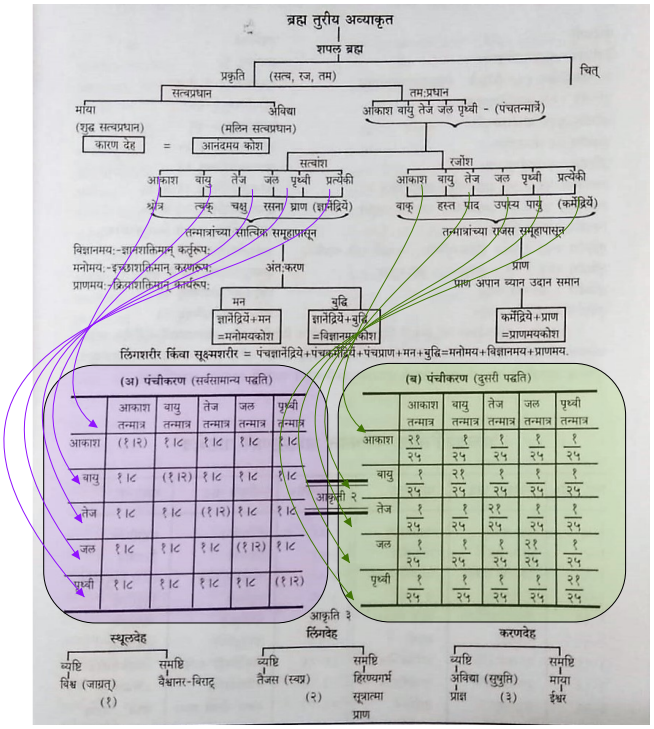


Figure 3.9: 'Pañcīkaraṇa' as per Pañcadaśī

हिरण्यगर्भः समवर्तताग्रे भूतस्य जातः पतिरेक आसीत् ।
स दधार पृथिवीं द्यामुतेमां कस्मै देवाय हविषा विधेम ॥ ऋग्वेद १०.१२१.१
Thus, Hiraṇyagarbha (हिरण्यगर्भ) was created first and then he created everything else.

- This Hiraṇyagarbha (हिरण्यगर्भ) gets manifested as Virāṭa Puruṣa (विराट पुरुष).
- यज्ञेन यज्ञमयजंत देवाः तानि धर्माणि प्रथमान्यासन् ।
  ते ह नाकं महिमानः सचंत यत्र पूर्वे साध्याः संति देवाः ॥ ऋग्वेद १०.९०.१६ वा १.१६४.१

- This १०.९०. is the well known 'PuruṣaSūkta' (पुरुषसूक्त)!
- The manifestation of the entire universe is crisply given in 'PuruṣaSūkta' (पुरुषसूक्त).
- सहस्रशीर्षा पुरुषः सहस्राक्षः सहस्रपात् ।
  स भूमिं विश्वतो वृत्वात्यतिष्ठद्दशांगुलम् ॥ऋग्वेद १०.९०.१
- ऋग्वेद १०.९०.१ can be understood as follows: That 'Puruṣa (पुरुष)' or the Universal form has thousand heads, eyes and feet (here thousand should be understood as innumerable). This 'Puruṣa (पुरुष)' encompasses and pervades every piece of creation and yet extends beyond the realms of the ten directions symbolized by ten fingers i.e. the entire universe!

3.3.4 Nāsadīya Sūkta - Verse Number 4

The fourth verse along with the anvaya (अन्वय) is as follows:

.. Ṛk No. 4 ..

.. ऋक् क्र. ४ ..

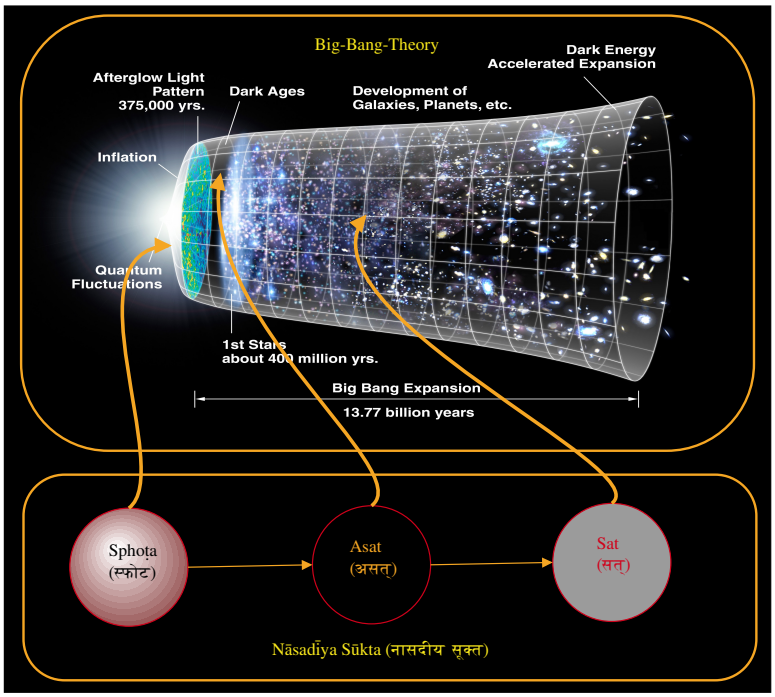


Figure 3.10: Primary Connectionism between The Big Bang and Nāsadīya Sūkta

- कामस्तदग्रे समवर्ततााधि मनसो रेतः प्रथमं तदासीत् ।
  सतो बन्धुमसति निरविन्दन्हृदि प्रतीष्या कवयो मनीषा ॥ १०.१२९.०४
- अन्वय - कामः । तत् । अग्रे । सं । अवर्तत । अधि । मनसः । रेतः । प्रथमं । यत् । आसीत् । सतः । बंधुं । असति । निः । अविंदन् । हृदि । प्रतिऽइष्य । कवयः । मनीषा ॥ १०-१२९-०४
- शब्दार्थ - कामः = radiance, perturbation । तत् = that । अग्रे = at the beginning of the world । सं = union, conjugation । अवर्तत = repeating, churning । अधि = foundation, reference place । मनसः = mental, intellectual । रेतः = seed, virile । प्रथमं = foremost । यत् = this । आसीत् = existed । सतः = manifested world, being । बंधुं = connect, relation । असति = at non-manifested, non-being । निः । अविंदन् = comprehended, figured out । हृदि = in heart, soul । प्रतिऽइष्य = seeking । कवयः = seers, sages, wise men । मनीषा ॥ = wisdom, intelligence १०-१२९-०४

Following Table (3.7) provides the views and interpretations of these 15 scholars on verse 4 of the present hymn.

Analytics 3.3.4 — Proposed decoding of Verse 4. This according to us is the most important verse to decipher. We will now look at every word and try and decipher it towards one unified interpretation. The attempt is to blend the spiritual view with the scientific view and show that

Comparative Interpretations of scholars on Verse 4 of Nāsadīya Sūkta	
Name of the interpreter	Interpretation
Vivekananda [27]	First desire rose, the primal seed of mind, (The sages have seen all this in their hearts - Sifting existence from non-existence). Its rays above, below and sideways spread.
Max Mueller [28]	Then first came Love upon it, the new spring of mind - yea, poets in their hearts discerned, pondering, this bond between created things and uncreated.
A. A. Macdonell [29]	Desire entered the One in the beginning: It was the earliest seed, of thought the product. The sages searching in their hearts with wisdom, Found out the bond of being in non-being.
Raimundo Panikkar [30]	In the beginning Love arose, which was primal germ cell of mind.The Seers, searching in their hearts with wisdom, discovered the connection of Being in Nonbeing
Ralph Griffith [31]	Thereafter rose Desire in the beginning, Desire, the primal seed and germ of Spirit. Sages who searched with their heart's thought discovered the existent's kinship in the non-existent.
Jamison & Brereton [32]	Then, in the beginning, from thought there developed desire, which existed as the primal semen. Searching in their hearts through inspired thinking, poets found the connection of the existent in the non-existent.
Geldner [33]	Uber dieses kam am Anfang das Liebesverlangen, was des Denkens erster Same war. Im Herzen forschend machten die Weisen durch Nachdenken das Band des Seins im Nichtsein ausfindig.
A. L. Basham [34]	In the beginning desire descended on it - that was the primal seed, born of the mind. The sages who have searched their hearts with wisdom know that which is, is kin to that which is not.
Wilson, HH: [35]	In the beginning there was desire, which was the first seed of mind; sages having meditated in their hearts have discovered by their wisdom the connexion of the existent with the non-existent.
Wendy Doniger O'Flaherty [36]	Desire came upon that one in the beginning; that was the first seed of mind. Poets seeking in their heart with wisdom found the bond of existence in non-existence.
Pandit RamGovind Trivedi [37]	सर्व-प्रथम परमात्मा के मन मे काम (सृष्टि की इच्छा) उत्पन्न हुआ। उससे सर्व-प्रथम बीज (उत्पत्तिकारण) निकला।बुद्धिमानो ने, बुद्धि के द्वारा, अपने अन्तःकरण मे विचार करके अविद्यमान वस्तु से विद्यमान वस्तु का उत्पत्ति-स्थान निरूपित किया।
Siddheshwar Shastri Chitrav [38]	ब्रह्माच्या मनोबीजातून सर्वप्रथम काम (सृष्टि निर्माणप्रवृत्ती) उत्पन्न झाली. मूळ परब्रह्माचा (असत्) विनाशी आणि दृश्य सृष्टीशी (सत्) जडलेला हा पहिला संबंध होय.
Krishnananda [39]	In the beginning desire descended on it that was the primal seed, born of the mind. The sages who have searched their hearts with wisdom know that which is is kin to that which is not
Hari Damodar Velankar [40]	उस सृष्टिपूर्व समयमे, मनोरूपी पुरुषका जो सर्वप्रथम (सृष्टिको निर्माण करनेवाला) काम (इच्छा) रूपी रेत उसने उस (परमात्वतत्त्व) का आश्रय ले लिया। प्रतिभासंपन्न कवियोने इस प्रकार अपनी प्रार्थनाके बलपर हृदयमे ही खोजकर इस वर्तमान (जगत्) के अव्याकृत परमात्मतत्त्वसे विद्यमान रहे हुए निकट संबन्धको निश्चितरूपसे जान लिया।
Sāyaṇācārya [41]	ननूक्तरीत्या यदीश्वरस्य पर्यालोचनं जगतः.....

Table 3.7: Contemporary Interpretations: Verse 4 of Nāsadīya Sūkta

they have huge overlap and commonalities.

Here, it should be noted that the first two verses (मंत्र) of Nāsadīya Sūkta (नासदीय सूक्त) discuss the situation at Pre-primordial times. Now, verses 3-4-5 will discuss the Primordial time. This is the time when the universe comes into existence and the process and sequence of the universe getting manifested is mentioned now. We would also like to mention that some of the critically acclaimed researchers like [43] have ignored this verse stating that this is not in compliance with the modern physics and science in general. This is indeed disheartening. We believe that this is the most crucial clue in decoding the complete hymn and hence can not be either ignored or neglected. Therefore, we present our analysis now.

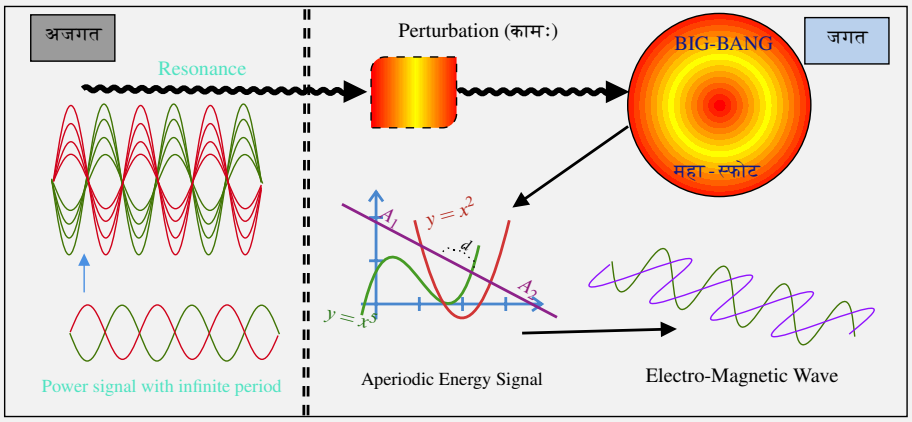
अग्रे:-
- This mention of time is different than the first two verses and similar to the one in verse 3. The words 'तदानीं' and 'तर्हि' from the first two verses indicate the time which is Pre-primordial. When nothing existed except that ONE.
- This word 'अग्रे' indicates the primordial time just like in verse 3. So this is the time of actual beginning of the universe.
- This is that time when everything is still unmanifested, however now that ONE is to become many!
- This is time just after the Big-Bang or महा - परिस्फोट.

यत् मनसः रेतः:- This refers to the mental seed of that ONE.
- Here word मनसः or mental essentially means the inherent exact overlap of nodes and antinodes in the standing wave.
- It must be noted that word रेतः can not be interpreted as semen or śukra (शुक्र).
- This is in our opinion gross error committed by many interpreters.
- In [26] for example, the author has misinterpreted this verse to such an extent that he has ended up writing a complete volume on sexology.
- This is phenomenon of creation of universe, truly extra-ordinary. Naturally, we can not apply the theorems of ordinary realms to such an extraordinary phenomenon.
- Here, रेतः only means the seed of the very creation in the making.
- This also lies in the vibrating and expanding nature of the standing wave.

प्रथमं आसीत्:- Initially existed. This is with reference to यत् मनसः रेतः. Thus, it together indicates that initially at the time of creation of the universe, first the mental seed existed and the rest followed.

तदधि कामः:- This is the moment of truth in our work! This is the most critical interpretation and if not done appropriately, then there is sanger of the complete work going on toss. Here, there are two terms, let's understand them:
- तदधि indicates the reference or the foundation for that. And 'that' here is मनसः रेतः or the mental seed.
- Now, comes the most critical one. Decoding and understanding the meaning of 'Kāmaḥ' (कामः).
- Here, 'Kāma' (काम) can simply not be the desire, let alone sexual desire of any kind.
- Most of the interpreters from Table (3.7) have used word 'desire'. Max Müller has used the word 'love'.
- Rajwade [26] has given the most unacceptable interpretation and has written complete volume on sexology.
- Let's decode it appropriately.
- We interpret 'Kāma' (काम) as radiance or perturbation.

- Please refer to the illustration given below.



- As shown, in the 'Ajagat' (अजगत्) stage, that ONE is a power signal with ∞ period.
- For such power signal to break it's periodicity, equally strong perturbation is needed.
- The resonance builds it up to effectuate it into a strong perturbation or probe, which is captured by the word 'Kāma' (काम).
- Thus, 'Kāma' (काम) means perturbation or probe or disturbance, which breaks the period and makes the original ONE signal aperiodic.
- The aperiodic signal is an Energy signal, which has finite energy and zero average power.
- The moment power signal becomes energy signal, 'Ajagat' (अजगत्) stage becomes 'Jagat' (जगत्) stage.
- 'Jagat' (जगत्) stage is thus manifestation of energy and it's propagation for transfer and change of mode.
- This is the reason why Yāska (यास्क) claims only three important deities, which are energy forms!
- 'Sūrya' (सूर्य), 'Indra' (इन्द्र) and 'Agni' (अग्नि) are all energy forms!
- That is why 'Ajagat' (अजगत्) stage was **Power** phase and 'Jagat' (जगत्) stage is **Energy** stage.
- A big enough probe is 'Kāma' (काम) which breaks the periodicity.
- Thus, 'Kāma' (काम) = the perturbation is the reason for breaking the periodicity, converting Power signal into it's Energy manifestation, converting 'Ajagat' (अजगत्) stage into 'Jagat' (जगत्) stage.
- In short, 'Kāma' (काम) = the probe is the most fundamental Nimitta Kāraṇa (निमित्त कारण = Incidental Cause) behind creation of this Universe.

सं अवर्तत**:-** This clarifies the above concept further. A doubt may arise here, how 'Kāma' (काम) = the perturbation gets generated? There has to be a logical explanation to this. The same is captured in सं अवर्तत = समवर्तत.

- Here word 'Saṃ' (सं) means conjugation or union
- 'Avartata' (अवर्तत) means repeatition or churning or gyration. E.g. सहस्रावर्तन is repeated recital of some hymn 1000 times.
- So, together 'Samavartata' (समवर्तत) essentially means सम्यगजायत. This is an adjec-

tive of 'Kāma' or the perturbation.

- 'Samavartata' (समवर्तत) indicates that the 'Kāma' or the perturbation got created out of the unison within the standing wave, thus leading to resonance, which finally got manifested into the giant probe which disrupted the periodicity of the Power Signal.
- This disruption resulted into the Big Bang or महा - परि - स्फोट.

Thus, the sequence of starting phase of creation of world (जगत्सृजनारंभ) is:

1. The ONE power entity with finite power and infinite energy.
2. Formation of standing wave with overlapping nodes and antinodes.
3. This leading to resonance and creation of 'Tapa' (तप) - 'स तपोऽतप्यत'.
4. This leads to strong perturbation converting power entity into an energy entity.
5. The energy entity has finite power - 'एकोऽहं बहुस्यां'.
6. 'Tapa' (तप) creating 'Salila' (सलिल).
7. From 'Salila' (सलिल) creation of the rest of it.

This concept of strong perturbation makes sense and can also be confirmed from second Kāṇḍa (द्वितीय काण्ड), second Prapāṭhaka (द्वितीय प्रपाठक), nineth Anuvāka (नवम् अनुवाक) of Taittirīya Brāhmaṇa (तैत्तिरीय ब्राह्मण) with word 'तदतप्यत'.

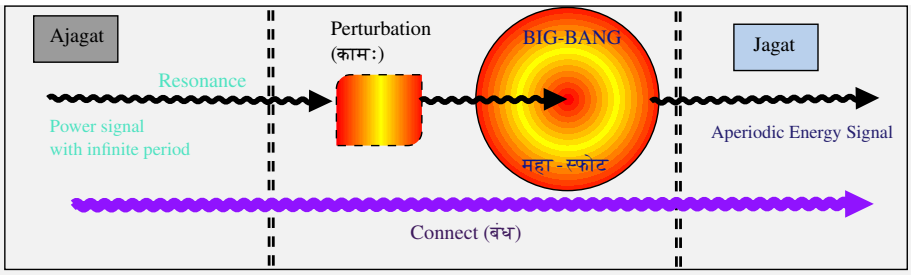
कवयः हृदि प्रतीष्य मनीषा निरविन्दन्:- The seers, after striving very hard, could confirm this wisdom in their hearts, souls and minds!

- Here, the word निरविन्दन् indicates that this is a discovery!
- No discovery happens without striving hard and putting the brains together. This is captured in प्रतीष्य.
- मनीषा is wisdom! This goes beyond just the integral knowledge or ज्ञान. This reconfirms, that this hymn is of highest quality and describes the most ultimate that is there.
- हृदि captures the hearts, minds and souls of these great Vedic researchers (वैदिक संशोधक).
- That is why they are called as कवयः. Kavī from root 'Kū' indicates wise person.

So what do these wise men understood as an eternal wisdom is:

असति सतः बंधुं:- The 'Kāma' (काम) = the perturbation is the first connect between Asat (असत्) world or non-existent world and Sat (सत्) world or existent world.

- Asat (असत्) world in the non-manifested world. This is the power signal. It has infinite energy which is unmanifested.
- For this Asat (असत्) to become Sat (सत्) or for the power signal to become energy signal, a connect is needed.
- This first connect which spans unmanifested power signal to manifested energy signal is 'Kāma' (काम) = the perturbation.
- The power signal has infinite energy but only exists in potentia and that is why it is unmanifested!
- This world is all about energy and energy transformations. Thus, this is the first connect that helped in manifestation.
- This is illustrated as follows:



Thus Verse-3 of Nāsadīya Sūkta (नासदीय सूक्त) can be summarized as follows:

अग्रे,	At Primordial time (beginning of universe);
यत् मनसः रेतः,	That mental seed of the ONE;
प्रथमं आसीत,	Which existed at the beginning
तदधि कामः,	is what became the foundation for;
	the radiance or the perturbation;
समवर्तत,	which existed in unison with the seed;
	eventually leading to resonance;
असति सतः बंधुं,	This Kāma (perturbation) is the;
	first connect between unmanifested and manifested;
कवयः हृदि प्रतीष्य,	After seeking and striving hard;
	in the hearts of the wise men;
मनीषा निरविन्दन्,	This wisdom was discovered.

Drawing correspondences: Verse 4

The question here is how Kāma can be interpreted as perturbation? Further to this, to break the ∞ periodicity of the original power signal, what ensures that the perturbation or the probe is large enough? The answer to this question is 'resonance'! This is neatly captured in समवर्तत!

- In modern science, resonance is a phenomenon in which a vibrating system gets driven to oscillate with greater amplitude at restricted range of frequencies
- The magnification in magnitude is closeness between system natural frequency and the external excitation frequency.
- These frequencies at which the response amplitude is highly magnified is known as 'resonant frequencies'.
- At such resonant frequencies, a small periodic force can also produce oscillations with huge amplitudes.
- This accounts to total vibrational energy.
- Let's recollect that Svāmī Vivekānanda has used the term 'vibration' to describe the extra-ordinary phenomenon captured in verse 2.
- Let ω_0 be the natural and ω_A be the input frequency.
- Here damping coefficient δ can be defined as:

$$\delta = 1 - (\frac{\omega_A}{\omega_0})^2 \tag{3.33}$$

- The resonance transmissibility is calculated as:

$$|G(\omega_A)| = \frac{1}{|1 - (\frac{\omega_A}{\omega_0})^2|} \tag{3.34}$$

- From equation (3.34), it is quite clear that when ω_0 matches with ω_A, it makes $\delta = 0$.
- When $\delta = 0$, the resonance transmissibility $|G(\omega_A)| = \infty$
- This phenomenon of $|G(\omega_A)| \to \infty$ is nothing else but The-Big-Bang or महा - परिस्फोट!
- This is neatly captured in Figure (3.11).

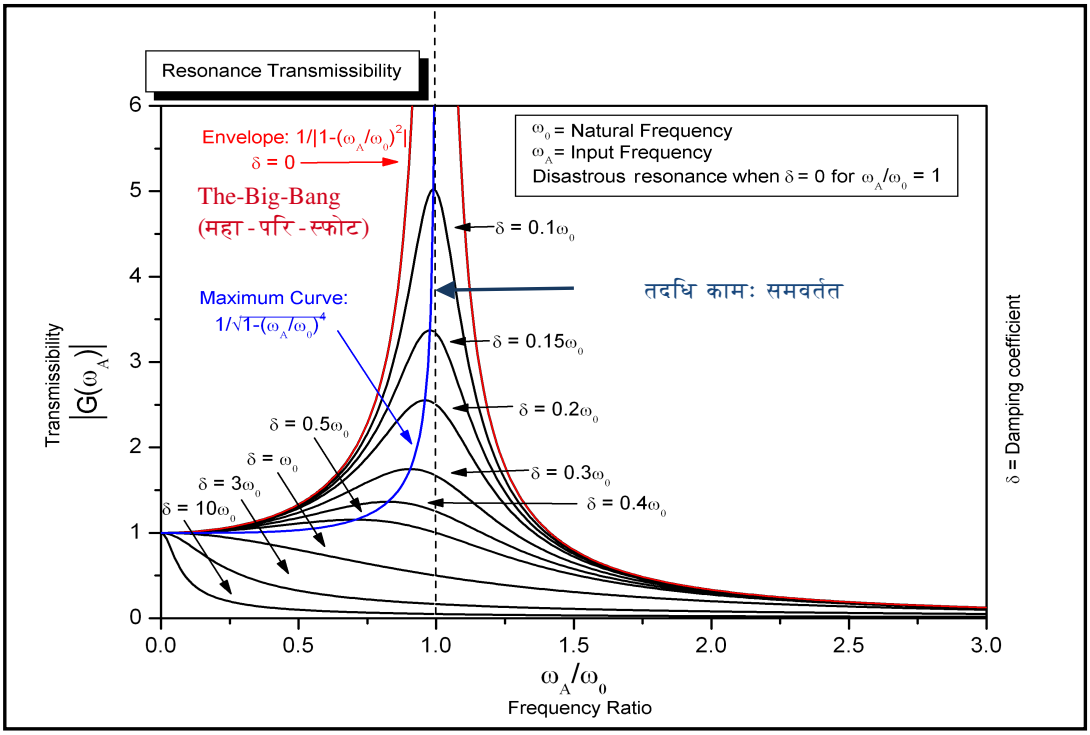


Figure 3.11: Resonance creating The-Big-Bang.

3.3.5 Nāsadīya Sūkta - Verse Number 5

The fifth verse along with the anvaya (अन्वय) is as follows:

.. Ṛk No. 5 ..

.. ऋक् क्र. ५ ..

- तिरश्चीनो विततो रश्मिरेषामधः स्विदासीऽदुपरि स्विदासीऽत् ।
  रेतोधा आसन्महिमान आसन्त्स्वधा अवस्तात्प्रयतिः परस्तात् ॥ १०.१२९.०५
- अन्वय - तिरश्चीनः । विऽततः । रश्मिः । एषां । अधः । स्वित् । आसीऽत् । उपरी । स्वित् । आसीऽत् । रेतःऽधाः । आसन् । महिमानः । आसन् । स्वधा । अवस्तात् । प्रयतिः । परस्तात् ॥ १०-१२९-०५

- शब्दार्थ - तिरश्चीनः = horizontal, across । विऽततः = extended, far-spread । रश्मिः = ray of light, cord, beam । एषां = seeking, following, going after । अधः । स्वित् । आसीऽत् = what (or indeed) existed below or beneath । उपरी । स्वित् । आसीऽत् = What (or indeed) existed above or upwards । रेतोऽधाः । आसन् = The seed bearing Prakṛti (प्रकृति) existed । महिमानः । आसन् = Might and power of Puruṣa (पुरुष) also existed । स्वधा = the inherent self-power । अवस्तात् = before । प्रयतिः । परस्तात् ॥ = Manifestation afterwards १०-१२९-०५

Following Table (3.8) provides the views and interpretations of these 15 scholars on verse 5 of the present hymn.

Analytics 3.3.5 — Proposed decoding of Verse 5. This verse provides the clues in an extremely subtle manner. This verse not only brings clarity on the necessary Dvaita (द्वैत) between Puruṣa (पुरुष) or Śapala Brahma (शपल ब्रह्म) and Prakṛti (प्रकृति) or Māyā (माया), but it also strengthens the doctrine of Satkāryavāda (सत्कार्यवाद).

Here, it should be noted that the first two verses (मंत्र) of Nāsadīya Sūkta (नासदीय सूक्त) discuss the situation at Pre-primordial times. Now, verses 3-4-5 will discuss the Primordial time. This is the time when the universe comes into existence and the process and sequence of the universe getting manifested is mentioned now.

तिरश्चीन्:- This should not only be interpreted as horizontal. We interpret this as transverse or one that goes beyond or across.

रश्मिः:- This means the ray of light, chord or the beam. This is that essential connect between the seed and the actual manifestation!

- The word Raśmi (रश्मि) is arguably the most important word of this verse.
- In the earlier verse that it was clearly mentioned that the perturbation eventuated into the seed which converted the power signal into an energy signal. This is essential for the further manifestation.
- In the previous verse it was mentioned: कामस्तदग्रे समवर्ततादि मनसो रेतः प्रथमं यदासीत्। Whose mind (मनस्) could this be? This has to be that of Hiraṇyagarbha (हिरण्यगर्भ). The journey from eternal to ephemeral happens through connect. This interesting connectionism is brought out by the word Raśmi (रश्मि).
- Carl Segan refers to Hiraṇyagarbha (हिरण्यगर्भ) as 'Cosmic-fire-ball'!
- From Table (3.8) it can be seen that various researchers have used different words to describe Raśmi (रश्मि) including: cord, ray of light, spark, धागा, बन्ध etc.
- This in a way confirms our hypothesis that the word in manifested through various forms of energy signals. For these signals to propagate the medium is essential. Word Raśmi (रश्मि) establishes that connect and thus confirms the transverse energy mapping.
- If स्वधया आनीत् अवातं is the cause behind The-Big-Bang (महा-परि-स्फोट), then the word Raśmi (रश्मि) actually captures the very phenomenon of The-Big-Bang (महा-परि-स्फोट).
- Raśmi (रश्मि) is clearly the indicator of The-Big-Bang (महा-परि-स्फोट) and hence it is not inappropriate to state that The-Big-Bang-Theory (महा-परि-स्फोट-सिद्धांत) was first stated clearly in Nāsadīya Sūkta.

Comparative Interpretations of scholars on Verse 5 of Nasadiya Sukta	
Name of the interpreter	Interpretation
Vivekananda [27]	Creative then became the glory, With self-sustaining principle below. And Creative Energy above.
Max Mueller [28]	Comes this spark from earth, piercing and all-pervading, or from heaven? Then seeds were sown, and mighty power arose - nature below, and Power and Will above.
A. A. Macdonell [29]	Their ray extended light across the darkness: But was the One above or was it under? Creative force was there, and fertile power: Below was energy, above was impulse.
Raimundo Panikkar [30]	A crosswise line cut Being from Nonbeing. What was described above it,what below?Bearers of seed there were and mighty forces, thrust from below and forward move above.
Ralph Griffith [31]	Transversely was their severing line extended : what was above it then, and what below it? There were begetters, there were mighty forces, free action here and energy up yonder.
Jamison & Brereton [32]	Their cord was stretched across: Did something exist below it? Did something exist above? There were placers of semen and there were powers. There was inherent force below, offering above.
Geldner [33]	Quer hindurch ward ihre Richtschnur gespannt, Gab es denn ein Unten, gab es denn ein Oben? Es waren Besamer, es waren Ausdehnungskräfte da. Unterhalb war der Trieb, oberhalb die Gewährung.
A. L. Basham [34]	And they have stretched their cord across the void, and know what was above, and what below. Seminal powers made fertile mighty forces. Below was strength, and over it was impulse.
Wilson, HH: [35]	Their ray was stretched out, whether across, or below, or above; (some) were shedders of seed, (others) were mighty; food was inferior, the eater was superior.
Wendy Doniger O'Flaherty [36]	Their cord was extended across. Was there below? Was there above? There were seed-placers; there were powers. There was impulse beneath; there was giving-forth above.
Pandit RamGovind Trivedi [37]	बीज-धारक पुरुष (भोक्ता) उत्पन्न हुए। महिलाये (भोग्य) उत्पन्न हुई। उन (भोक्ताओ) का कार्य -कलाप दोनो पार्श्वो (नीचे और ऊपर) विस्तृत हुआ। नीचे स्वधा (अन्न) सहा और ऊपर प्रयति (भोक्ता) अवस्थित हुआ।
Siddheshwar Shastri Chitrav [38]	सत् आणि असत् यामधील हा धागा (रश्मि) खालपासून वरपर्यंत आडवा पसरला. त्या धाग्यापैकी काही भाग बीजप्रद (रेतोधा) होऊन मोठा झाला. या धाग्याचीच स्वशक्ति अलिकडे आणि प्रभाव (प्रयति) पलीकडे पसरला.
Krishnananda [39]	And they have stretched their cord across the void, and know what was above, and what below. Seminal powers made fertile mighty forces. Below was strength, and over it was impulse.
Hari Damodar Velankar [40]	इन्हे जोडनेवाला बन्ध तिर्यक् होकर सर्वत्र फैला हुआ था। क्या यह (परमात्मतत्त्व) नीचे भी और ऊपर भी रहा था? (वह सचमुच दो भागोमे विभाजित हो गया और उसकी आधी शक्तिया) रेत:सेक करनेवाला पुरुषतत्त्व बन गैइ और शेष आधी (गर्भधारण - पोषण करने योग्य) विशाल (स्त्रीतत्त्व) हो गैइ। स्वधा शक्ति नीचे थी और प्रयतिशक्ति उसके पार (ऊपर) थी।
Sāyaṇācārya [41]	एवमविद्याकामकर्माणि सृष्टेर्हेतुत्वेनोक्तानि।.....

Table 3.8: Contemporary Interpretations: Verse 5 of Nāsadīya Sūkta

- Carl Segan mentions this Raśmi (रश्मि) as 'Fibre of Space'! It is baffling to see the similarity.

वितत्**::-** This refers to Raśmi (रश्मि) being extended or spread out or diffused. This it clearly states the diffusion of the solar rays as energy transverse mechanism.

एषां**:-** This indicates the act of seeking again in connection with the energy transcendence.

अधः स्वित् आसीऽत्**:-** This means What (or indeed) existed below or beneath.

उपरी स्वित् आसीऽत्**:-** This means What (or indeed) existed above or upwards.

- The above terms describe the scope and extent of Raśmi (रश्मि).
- This same cord or धागा indeed exists above as well as below.
- The words अधः or below and उपरी or above should also be understood from the perspective of Uttānapāda (उत्तानपाद) or Aśvatthavṛkṣa (अश्वत्थवृक्ष).
- For this Aśvatthavṛkṣa (अश्वत्थवृक्ष), the roots or the cause is above and the expansion or the effect is below.
- It is then this Raśmi (रश्मि) that connects the two and formalizes the cause-effect-relationship (कार्य - कारण - भाव).

रेतोऽधाः आसन्**:-** This means the seed bearing (बीजप्रद) Prakṛti (प्रकृति) or Māyā (माया) existed.

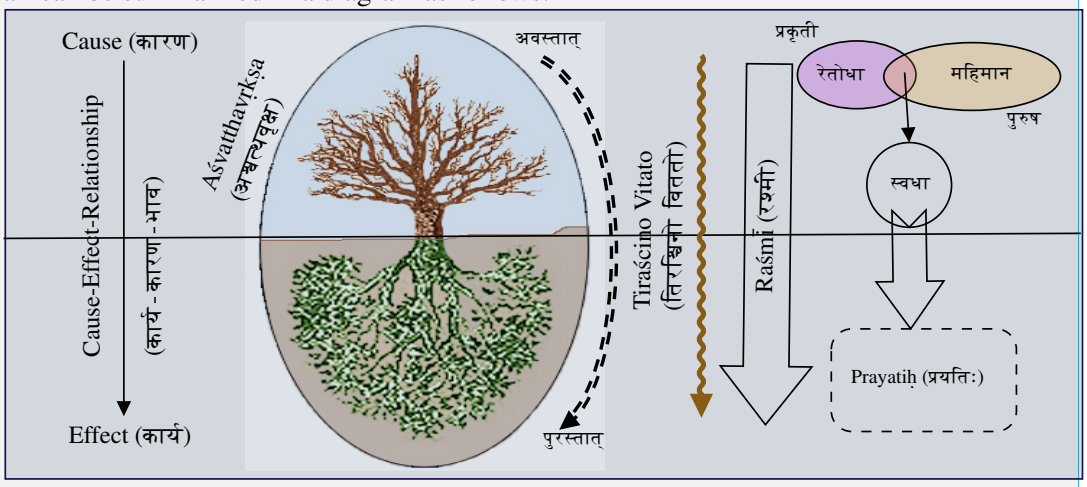
महिमानः आसन्**:-** This means Might and power of Puruṣa (पुरुष) or Śapala Brahma (शपल ब्रह्म) also existed.

- The above two terms describe exactly how the effect (कर्य) in the form of the universe got manifested.
- For the Aśvatthavṛkṣa (अश्वत्थवृक्ष) mentioned above, the roots are in the form of the seed which is captured by मनसो रेतः in the earlier verse.
- From these roots how the universe gets created is the question!
- This important question is answered using the expansion of the same themology mentioned indirectly in the first two verses. This themology is of:
  1. Dvaita (द्वैत) in the Antarikṣa (अन्तरिक्ष) - complementary dual
  2. Dvandva (द्वन्द्व) in the Pārthiva (पार्थिव) - contradictory duel
- The Dvaita (द्वैत) or the complementary dual needs to be bound by the connecting link, the Raśmi (रश्मि) again!
- The part that becomes रेतोऽधाः is the Prakṛti (प्रकृति) or Māyā (माया). From the signal processing perspective, we call this as the mother Wavelet function or $\psi(.)$ function.
- The part that becomes महिमानः is the Puruṣa (पुरुष) or Śapala Brahma (शपल ब्रह्म). From the signal processing perspective, we call this as the father Scaling function or $\phi(.)$ function.
- This will be explained in detail in the subsequent chapters in the context of Satkāryavāda (सत्कार्यवाद), starting with the robust Sāṁkhya (सांख्य) philosophy, ultimately culminating into Advaita Vedānta (अद्वैत वेदान्त) philosophy.

स्वधा अवस्तात् प्रयतिः परस्तात्**:-** This clearly states that the inherent self-kernel existed before and that created the rest of the manifestation. This स्वधा is sometimes what is popularly given nomenclature as 'Brahma-Deva' (ब्रह्म - देव) in the literature. From there starts the manifestation of the universe.

- Here स्वधा is the inherent self-power or the self-kernel! This is also explained from its mention in the previous verse.
- Sāyaṇācārya (सायणाचार्य) has given Mīmāṁsā (मीमांसा) as: 'अन्ननामैतत् भोग्यप्रपंचः' [a]
- The word प्रयति captures the multi-fold manifestation!

This all can be summarized in a diagram as follows:



Here we end our direct interpretation of verse 5 of Nāsadīya Sūkta by summarizing it.

Thus Verse-5 of Nāsadīya Sūkta (नासदीय सूक्त) can be summarized as follows:

तिरश्चीनो वितत्:,	Across got extended;
रश्मिरेषाम्,	the seeking cord or the ray of light;
अधः स्वित् आसीऽत्,	This is what (or indeed) existed below or beneath
उपरी स्वित् आसीऽत्,	This is what (or indeed) existed above or upwards;
रेतोऽधाः आसन्,	The seed bearing Prakṛti (प्रकृति); or Māyā (माया) existed;
महिमानः आसन्,	Might and power of Puruṣa (पुरुष); or Śapala Brahma (शपल ब्रह्म) also existed;
स्वधा अवस्तात्,	The inherent self-power or self-kernel; existed before (in time);
प्रयतिः परस्तात्,	The Universe Manifested afterwards

[a]तैत्तिरीय आरण्यक ८.६

Drawing correspondences: Verse 5

From the perspective of drawing correspondences, this verse is the most important one according to us. In fact, this verse takes us on the lines proposed by one of the greatest minds of the last century, Albert Einstein!

Einstein was trying to figure out, in this constantly changing world (परिवर्तनीय संसारे.... as per भर्तृहरी), what is it that remains constant or unchanged? After years of observations, cross-validating those with the laws of physics and by virtue of performing several experiments, he came up with interesting answer. According to Einstein, in this constantly expanding and changing world, what remains unchanged is the speed of light! The speed with which the light rays or Raśmi (रश्मि) travels is what remains constant. Therefore, the energy mapping in the universe, the transverse mass-energy conversion and relative moments can only be mapped by keeping Raśmi (रश्मि) as the central theme. Is it not mind blowing, that Ṛṣi Parameṣṭhī (ऋषि परमेष्ठी) already knew this! This great sage was not only able to envisage this, but he was also able to put this in practise! In this verse he clearly states that the same sun-rays transcend and are responsible for mass-energy conversion, conservation and balance. In modern era, we call this as 'The Theory of Special Relativity - by Albert Einstein'!

Theory of Special Relativity by Albert Einstein:

- Year 1905 is probably the most interesting year in the history of modern physics.
- Before Einstein 'time' and 'space' were considered to be distinct, uncorrelated and separate.
- Similarly, before Einstein 'mass' and 'energy' were also considered to be distinct, uncorrelated and separate.
- By bringing these then seemingly uncorrelated entities together, Einstein could carve out a niché!
- First the concept of 4D space-time was phenomenal.
- On the top, by connecting energy and mass with famous equation $E = mc^2$, Einstein could complete his theory of special relativity.

Calculations 3.2 — The famous $E = mc^2$. In 1905, when Einstein proposed his Special Theory of Relativity and introduced $E = mc^2$, very few people could comprehend it. In that sense, Einstein was truly ahead of his time. It was only when in 1919, series of experiments were carried out at Geneva and the theory was finally proved by inference. We will see the quick intuitive proof of this famous equation and connect it with verse 5 of Nāsadīya Sūkta.

- The crux of this proof lies in the principle of the constancy of the speed of light.
- The speed of light in a vacuum has the same constant value c.
- This c of Einstein is captured by Raśmi (रश्मि) in this verse 5 of Nāsadīya Sūkta.
- $E = mc^2$ is derived directly from Special Relativity.
- Because of c, mass can be turned into Energy. This is the essence of the manifestation and expansion of the Universe.
- उपरी स्वित् आसीऽत् = Energy E.
- अधः स्वित् आसीऽत् = mass m.
- One was of understanding the Special Relativity is the mass changes with speed!
- In our day to day activities the speed is so low, that the mass change goes unnoticed, it exists though!
- The following equation specifies by how much mass increases with reference to speed:

$$m = \frac{m_0}{\sqrt{(1 - v^2)/c^2}} \tag{3.35}$$

where,
$$\begin{cases} m_0, & \text{mass of stationary object;} \\ m, & \text{mass of moving object;} \\ v, & \text{speed of velocity of object;} \\ c, & \text{speed of light.} \end{cases}$$

- Thus, as an objects travels faster, its mass appears to increase
- The kinetic energy of moving objects is give by,

$$KE = \frac{1}{2}mv^2 \tag{3.36}$$

- Now, if v is very low in equation (3.35), then increase in the mass gets accounted by $\approx mc^2$.
- The total energy, taking into account the Kinetic Energy and mass change can be written as

$$E \approx mc^2 + \frac{1}{2}mv^2 \tag{3.37}$$

- For any body with zero speed, the equation reduces to $E = mc^2$ as the Relative Kinetic Energy reduces to zero.
- Thus, for any non-stationary body, the total energy will be given as,

$$m = \frac{mc^2}{\sqrt{(1-v^2)/c^2}} \tag{3.38}$$

- This is an equation of an Energy and talks about the energy with potential in the form of mass m.
- Nuclear atomic stations, Nuclear weapons are the best examples to see this equation is use.

■

3.3.6 Nāsadīya Sūkta - Verse Number 6

The sixth verse along with the anvaya (अन्वय) is as follows:

.. Rk No. 6 ..

.. ऋक् क्र. ६ ..

- को अद्धा वेद क इह प्र वोचत्कुत आजाता कुत इयं विसृष्टिः ।
  अर्वाग्देवा अस्य विसर्जनेनाथा को वेद यत आबभूव ॥ १०.१२९.०६
- अन्वय - को । अद्धा । वेद । क । इह । प्र । वोचत । कुत । आजाता । कुत । इयम् । विसृष्टिः । अर्वाग । देवा । अस्य । विसर्जने । ना । आथा । को । वेद । यत । आब । भूव ॥ १०-१२९-०६
- शब्दार्थ - को = who, what । अद्धा = certainly । वेद = knows । क = who, what । इह = here । प्र = about which । वोचत = being spoken । कुत = how, whence । आजाता = produced । कुत = how, whence । इयम् = this । विसृष्टिः = Multifold manifestation । अर्वाग = after । देवा = Gods । अस्य = abode, becoming । विसर्जने = creation । ना = no? । आथा = Then । को = who, what । वेद = to know । यत = the place । आब = beginning । भूव ॥ = to come into being १०-१२९-०६

Following Table (3.9) provides the views and interpretations of these 15 scholars on verse 6 of the present hymn.

Analytics 3.3.6 — Proposed decoding of Verse 6. This verse most certainly pushes the entire discussion towards Advait Vedānta philosophy (अद्वैत वेदान्त तत्त्वज्ञान).

Here, it should be noted that the first two verses (मंत्र) of Nāsadīya Sūkta (नासदीय सूक्त) discuss the situation at Pre-primordial times. Then, verses 3-4-5 will discuss the Primordial time. Now,

Comparative Interpretations of scholars on Verse 6 of Nasadiya Sukta	
Name of the interpreter	Interpretation
Vivekananda [27]	Who knew the way? Who there declared Whence this arose? Projection whence? For after this projection came the gods. Who therefore knew indeed, came out this whence?
Max Mueller [28]	Who knows the secret? who proclaimed it here, whence, whence this manifold creation sprang? - The gods themselves came later into being. - Who knows from whence this great creation sprang? -
A. A. Macdonell [29]	Who knows for certain? Who shall here declare it? Whence was it born, and whence came this creation? The gods were born after this world's creation: Then who can know from whence it has arisen?
Raimundo Panikkar [30]	Who really knows? Who can presume to tell it? Whence was it born? Whence issued this creation?Even the Gods came after its emergence. Then who can tell from whence it came to be?
Ralph Griffith [31]	Who verily knows and who can here declare it, whence it was born and whence comes this creation? The Gods are later than this world's production. Who knows then whence it first came into being?
Jamison & Brereton [32]	Who really knows? Who shall here proclaim it? - from where was it born, from where this creation? The gods are on this side of the creation of this world. So then who does know from where it came to be?
Geldner [33]	Wer weiss es gewiss, wer kann es hier verkünden, woher sie entstanden, woher diese Schöpfung kam? Die Götter kamen erst nachher durch die Schöpfung dieser Welt. Wer weiss es dann, woraus sie sich entwickelt hat?
A. L. Basham [34]	But, after all, who knows, and who can say whence it all came, and how creation happened? The gods themselves are later than creation, so who knows truly whence it has arisen?
Wilson, HH: [35]	Who really knows? who in this world may declare it? whence was this creation, whence was it engendered? The gods (were) subsequent to the (world's) creation; so who knows whence it arose?
Wendy Doniger O'Flaherty [36]	Who really knows? Who will here proclaim it? Whence was it produced? Whence is this creation? The gods came afterwards, with the creation of this universe. Who then knows whence it has arisen?
Pandit RamGovind Trivedi [37]	प्रकृत तत्त्व को कौन जानता है? कौन उसका वर्णन करे? यह सृष्टि किस उपादान कारण से हुई? किस निमित्त कारण से ये विविध सृष्टिया हुई? देवता लोग इन सृष्टियो के अनन्तर उत्पन्न हुए है। कहा से सृष्टि हुई, यह कौन जानता है?
Siddheshwar Shastri Chitrav [38]	सत् चा हा पसरा (विसर्ग) कशापासून आणि कोठून उत्पन्न झाला, हे यापेक्षा अधिक विस्ताराने (प्र) कोण सांगू शकेल? सत् सृष्टिच्या विसर्गानंतर उत्पन्न झालेल्या देवांनाहि हे ज्ञात नाही. मग सृष्टी कोठून निघाली, हे कोण जाणणार?
Krishnananda [39]	But, after all, who knows, and who can say Whence it all came, and how creation happened? the gods themselves are later than creation, so who knows truly whence it has arisen?
Hari Damodar Velankar [40]	सचमुच यह कौन जानता है? यहा कौन यह स्पष्ट रूपमे बता सकेगा? यह समूची सृष्टि किससे उत्पन्न हुई और कहासे आई? ये देव इस जगतके निर्माणके उपरान्त अर्वाचीन समयके ही है। फिर तो, जिससे यह सृष्टि निर्माण हुई उसे किसने जान लिया है?
Sāyaṇācārya [41]	एवं भोक्तृभोग्यरूपेण.....

Table 3.9: Contemporary Interpretations: Verse 6 of Nāsadīya Sūkta

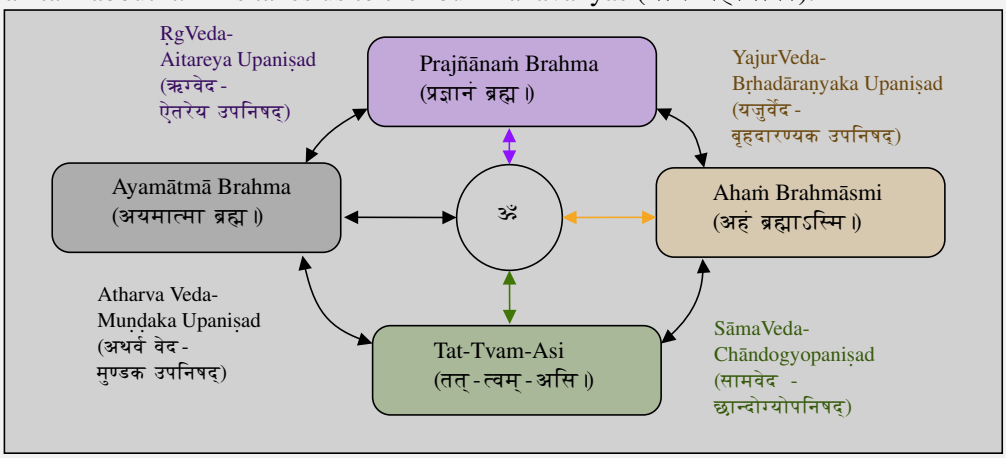
verses 6-7 raises interesting questions and builds intuitional model of the universe.

को अद्धा वेद कः- This means, who really knows or comprehends? A very interesting question is raised here. The questions leads our interpretation towards 'Ātma-Jñāna' (आत्म - ज्ञान) or self-realization!

- Know it, to know it.
- No it, then no it.

One who understands and comprehends the self-soul (जीवात्मा) and realizes that it is no different than the Supreme-soul (परमात्मा) only knows! This is the crux of Advait Vedānta philosophy (अद्वैत वेदान्त तत्त्वज्ञान).

क इह प्रवोचत:- This means who can talk about it? Well, only the one who knows and comprehends can talk about it. This takes us to the four Mahāvākyas (चार महावाक्ये).



- Prajñānaṁ Brahma (प्रज्ञानं ब्रह्म ।) appears in ṚgVeda - Aitareya Upaniṣad - (3.3) (ऋग्वेद - ऐतरेय उपनिषद्). It is the concept of that ONE becoming many. The first two chapters of ṚgVeda- Aitareya Upaniṣad (ऋग्वेद - ऐतरेय उपनिषद्) talk about the creator and multi-fold manifestation of the same soul.
- Aham Brahmāsmi (अहं ब्रह्माऽस्मि ।) appears in the YajurVeda- Bṛhadāraṇyaka Upaniṣad- (1.410) (यजुर्वेद - बृहदारण्यक उपनिषद्). This establishes the true nature of Brahman through dialogue between Gārgya (गार्ग्य) and Ajātaśatrū (अजातशत्रू).
- Tat-Tvam-Asi (तत् - त्वम् - असि ।) appears in SāmaVeda - Chāndogyopaniṣad - (6.8.7) (सामवेद - छान्दोग्योपनिषद्). It is the ultimate knowledge given by Uddālaka (उद्दालक) to his son Śvetaketū (श्वेतकेतू).
- Ayamātmā Brahma (अयमात्मा ब्रह्म ।) appears in Atharva Veda - Muṇḍaka Upaniṣad - (1.2) (अथर्व वेद - मुण्डक उपनिषद्). Upon insistence of Maharṣi Vyāsa (महर्षि व्यास), his son Śuka (शुक) gets this preaching from King Janaka (राजा जनक). This philosophy emphasizes on being Brahma-Conscious always.
- This is the very essence of Upaniṣadas (उपनिषद्:).
- These four statements are the ultimate truth, per say.

कुत आजाता:- This means who is the creator of this universe? This question has been answered in depth already, let's quickly recap that.

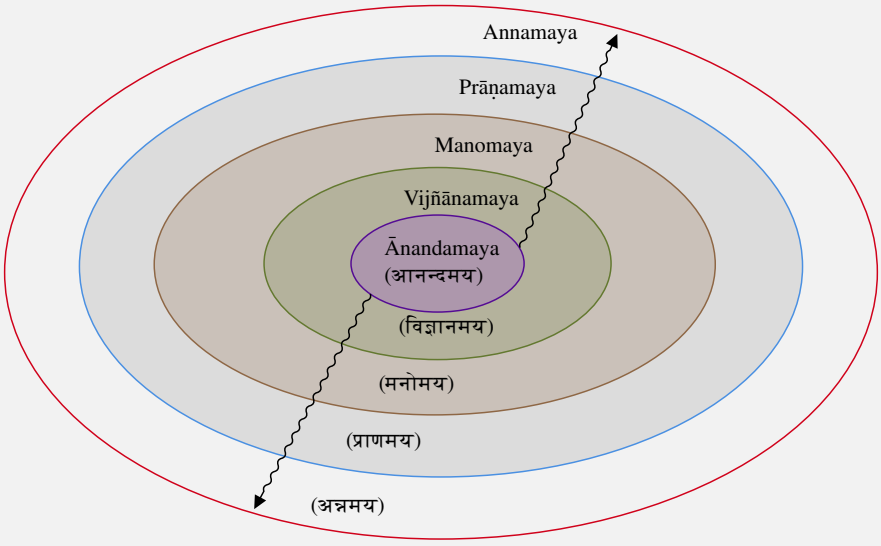
- That one which is referred to be creator of the universe is called upon by various named.
- 'Prajāpati' (प्रजापति), 'Ka' (क देवता), 'Hiraṇyagarbha' (हिरण्यगर्भ), 'Brahmadeva'

(ब्रह्मदेव) are the various names of it.

- This creator of the universe is also the one who sustains it and eventually destroys it for the next new creation.
- Saint Jñāneśvara that is why says 'सर्व सुखाचे आगरु बाप रखुमादेवीवरु।' [a].
- This creator has self-inherent-kernel-power captured by 'Svadhā' (स्वधा).
- This ultimate is called as 'Parabrahma' (परब्रह्म) or 'Paramātmā' (परमात्मा).
- The nature of this ultimate is captured by the शांतिमंत्र of YajurVeda- Bṛhadāraṇyaka Upaniṣad (यजुर्वेद - बृहदारण्यक उपनिषद्):
- पूर्णमदः पूर्णमिदं पूर्णात्पूर्णमुदच्यते। पूर्णस्य पूर्णमादाय पूर्णमेवावशिष्यते॥
- This Almighty is called as ईश as per Īśāvāsya Upaniṣada (ईशावास्य उपनिषद).
- ईशावास्यमिदं सर्वं यत्किंच जगत्यां जगत्। तेन त्यक्तेन भुंजीथा मा गृधः कस्य स्विद्धनम्॥१.१
- Here, ईशावास्य = ईश् + आवास्यं. This, Ultimate spans the universe inside out, in its entirety!

कुत इयं विसृष्टिः:- This means how the multi-fold manifestation of the universe took place from that one? This is an extremely important question and once again it indicates the framework proposed by the Advait school of thinking. The 'Sarga' (सर्ग) as per Advaita Vedānta (अद्वैत वेदान्त) philosophy is extremely comprehensive, all-encompassing and seamless. The essence of that can be expressed as shown in the Table (3.10) below.

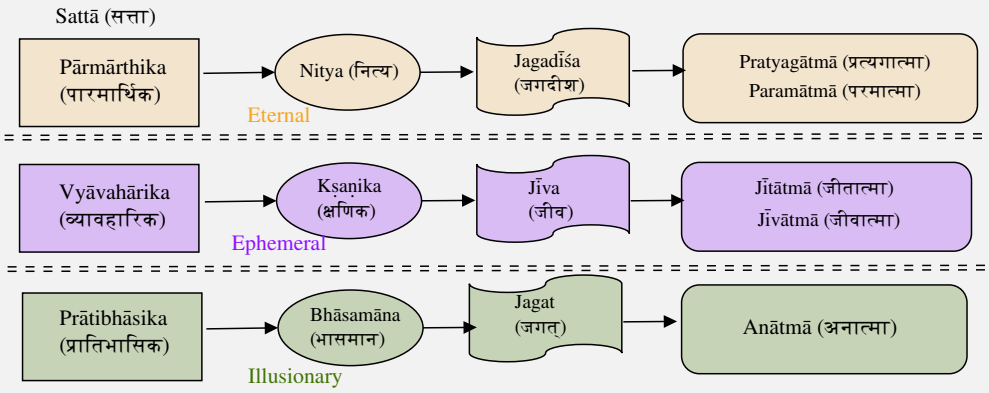
- In the seed form (बीज रूप), the Śarīra (शरीर) is Kāraṇa (कारण), Avasthā (अवस्था) is Suṣupti (सुषुप्ति) and Koṣa (कोष) is Ānandamaya (आनन्दमय).
- In this seed form, the being at individual level (Vyaṣṭī (व्यष्टी)) is Prājña (प्राज्ञ).
- Also in this seed form, the being at universal level (Samaṣṭī (समष्टी)) is Īśvar (ईश्वर).
- Similarly, all the forms and corresponding multi-level manifestation can be understood.
- The five level Koṣa (कोष) system is as depicted in the diagram below:



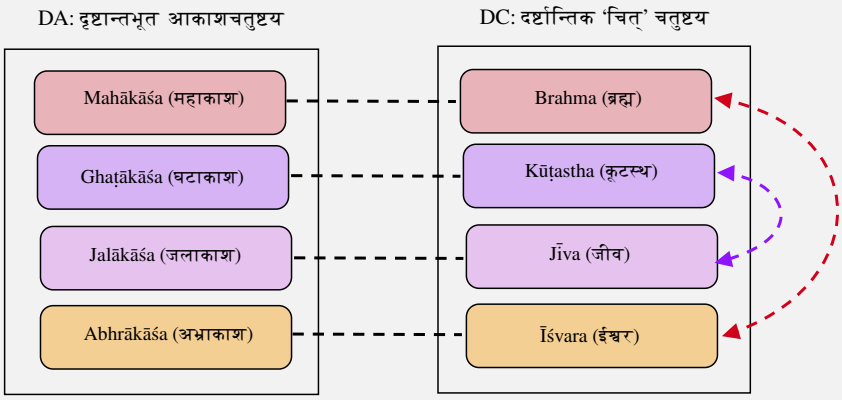
अर्वाग्देवा अस्य विसर्जनायः:- This clearly states that 'Deva' (देव = Gods) are born much after the creation and multi-fold manifestation of the universe!

अथा को वेद यत आबभूवः:- Therefore (as the Gods are much after the Universe), who really understand the journey from beginning (आब) to coming into being (भूव)?

- This verse very clearly states the temporal sequence of evolution.
- The above question is extremely critical and we again need to take help from Advait Vedānta philosophy (अद्वैत वेदान्त तत्त्वज्ञान) to uncover this.
- First we need to understand that this is undoubtedly 'Satkāryavāda' (सत्कार्यवाद) and there is no element of PariṇāmaVāda (परिणामवाद).
- This is because the Jīva (जीव), Jagat (जगत्) and Jagadīśa (जगदीश) - they all belong to different states of Sattā (सत्ता) as per Sattā-Traya (सत्ता - त्रय) doctrine of Ādi Śaṅkarācārya (आदि शंकराचार्य).
- This Sattā-Traya (सत्ता - त्रय) doctrine is depicted in diagram below:



- The question arises here: Why this verse explicitly mentions that Deva (देव) were born much after and they can not know how the universe came into existence?
- The answer of this important question lies in Advait Vedānta philosophy (अद्वैत वेदान्त तत्त्वज्ञान) yet again.
- This can be understood from the Anyonyādhyāsa (अन्योन्याध्यास) as given below:



Here we end our direct interpretation of verse 6 of Nāsadīya Sūkta by summarizing it.

Thus Verse-6 of Nāsadīya Sūkta (नासदीय सूक्त) can be summarized as follows:

को अद्धा वेद क,	who really knows or comprehends?;
क इह प्रवोचत,	who can talk about it?;
कुत आजाता,	who is the creator of this universe?
कुत इयं विसृष्टिः,	how the multi-fold manifestation;
	of the universe took place;
अर्वाग्देवा अस्य विसर्जनाय,	'Deva' (देव = Gods) are born;
	much after the creation and multi-fold manifestation;
अथा को वेद यत आबभूव,	Therefore, who really understand the journey;
	from beginning to coming into being?

[a]रूपाचा अभंग by संत ज्ञानेश्वर

yāsa (व्यास महर्षि)

(Indian) is also referred as Veda-Vyāsa (वेद - व्यास) for his contribution in structuring and classifying the Vedas. Because of his dark complexion, he is also referred as Kṛṣṇa Dvaipāyana (कृष्ण द्वैपायन). He was son of Parāśara (पराशर) and Satyavati (सत्यवति). He created so much literature, that the saying became popular - व्यासोच्छिष्टं जगत् सर्वम् ।

Form (रूप)	**Symbol** (प्रतिक)	**Śarīra** (शरीर)	**Avasthā** (अवस्था)	**Koṣa** (कोष)	**Vyaṣṭī** (व्यष्टी)	**Samaṣṭī** (समष्टी)
Bīja (बीज)	○	Kāraṇa (कारण)	Suṣupti (सुषुप्ति)	Ānandamaya (आनन्दमय)	Prājña (प्राज्ञ)	Īśvara (ईश्वर)
Aṃkūra (अंकूर)	⊖	Sūkṣma (सूक्ष्म)	Svapna (स्वप्न)	Vijñānamaya (विज्ञानमय)	Taijasa (तैजस)	Sūtrātmā (सूत्रात्मा) or Hiraṇyagarbha (हिरण्यगर्भ)
				Manomaya (मनोमय)		
				Prāṇamaya (प्राणमय)		
Ropa (रोप)	⩚	Sthūla (स्थूल)	Jāgṛti (जागृति)	Annamaya (अन्नमय)	Vaiśvānara (वैश्वानर)	Virāṭa (विराट)

Table 3.10: World Manifestation as per Advaita Vedānta

Drawing correspondences: Verse 6

- This clearly indicates that the initial condition are extremely important to determine the future behavior.

- The 'Deva' did not know the initial conditions and therefore, they may not be able to model the initial states.
- This clearly suggest a strong correlation with the 'Chaos Theory'.
- Within the apparent randomness, the complex chaotic pattern unfold. One way of capturing this is through the self-similarity. In the subsequent chapter we will show the self-similar structures, useful in such mapping.
- Conventionally, scientists believed that all the natural processes can either be deterministic or non-deterministic.
- The third category is needed, when small change in initial condition may bring out catastrophic change in the prediction model.
- Chaotic behavior exists in many natural systems, like weather and climate for example.
- The definition is give in [[18]] as follows: "In continuous time dynamical systems, chaos is the phenomenon of the spontaneous breakdown of topological supersymmetry which is an intrinsic property of evolution operators of all stochastic and deterministic (partial) differential equations."
- Mathematically it is captured as follows:

$$|\delta Z(t)| \approx e^{\lambda t} |\delta Z_0| \tag{3.39}$$

- In Equation (3.39), t = time, λ = Lyapunov exponent.
- The number of Lyapunov exponents is equal to the number of dimensions of the phase space.

3.3.7 Nāsadīya Sūkta - Verse Number 7

The seventh verse along with the anvaya (अन्वय) is as follows:

.. Rk No. 7 ..

.. ऋक् क्र. ७ ..

- इयं विसृष्टिर्यत आबभूव यदि वा दधे यदि वा न ।
  यो अस्याध्यक्षः परमे व्योमन्सो अङ्ग वेद यदि वा न वेद ॥ १०.१२९.०७
- अन्वय - इयं । विसृष्टिः । यत । आब । भूव । यदि । वा । दधे । यदि । वा । न । यो । अस्य । अध्यक्षः । परमे । व्योमन् । सो । अङ्ग । वेद । यदि । वा । न । वेद ॥ १०-१२९-०७
- शब्दार्थ - इयं = this । विसृष्टिः = Multifold manifestation । यत = where । आब । भूव = beginning to come into being । यदि । वा = whether-or । दधे = to possess । यदि । वा । न = whether-or-not । यो । अस्य = since, the abode । अध्यक्षः = power to perceive । परमे = highest । व्योमन् = heaven । सो = to conclude । अङ्ग = Indeed । वेद = to know । यदि । वा । न । वेद ॥ = whether-or-not-known १०-१२९-०७

Following Table (3.11) provides the views and interpretations of these 15 scholars on verse 7 of the present hymn.

Comparative Interpretations of scholars on Verse 7 of Nasadiya Sukta	
Name of the interpreter	Interpretation
Vivekananda [27]	This projection whence arose, Whether held or whether not, He the ruler in the supreme sky, of this He, O Sharman! knows, or knows not He perchance!
Max Mueller [28]	He from whom all this great creation came. Whether his will created or was mute, the Most High seer that is in highest heaven, he knows it, - or perchance e'en He knows not.
A. A. Macdonell [29]	None knoweth whence creation has arisen; And whether he has or has not produced it; He who surveys it in the highest heaven, He only knows, or haply he may know not.
Raimundo Panikkar [30]	That out of which creation has arisen, whether it held it firm or it did not,He who surveys it in the highest heaven, He surely knows - or maybe He does not!
Ralph Griffith [31]	He, the first origin of this creation, whether he formed it all or did not form it, whose eye controls this world in highest heaven, he verily knows it, or perhaps he knows not.
Jamison & Brereton [32]	This creation - from where it came to be, if it was produced or if not - he who is the overseer of this world in the highest heaven, he surely knows. Or if he does not know... ?
Geldner [33]	Woraus diese Schopfung sich entwickelt hat, ob er sie gemacht hat oder nicht - der der Aufseher dieser Welt im hochsten Himmel ist, der allein weiss es, es sei denn, dass auch er es nicht weiss.
A. L. Basham [34]	Whence all creation had its origin, he, whether he fashioned it or whether he did not, he, who surveys it all from highest heaven, he knows - or maybe even he does not know.
Wilson, HH: [35]	He from whom this creation arose, he may uphold it, or he may not (no one else can); he who is its superintendent in the highest heaven, he assuredly knows, or if he knows not (no one else does).
Wendy Doniger O'Flaherty [36]	Whence this creation has arisen - perhaps it formed itself, or perhaps it did not - the one who looks down on it, in the highest heaven, only he knows - or perhaps he does not know.
Pandit RamGovind Trivedi [37]	ये नाना सृष्टिया कहा से हुई, किसने सृष्टिया कि, और किसने नही की - यह सब वे ही जाने, जो इनके स्वामी परम धाम मे रहते है। हो सकता है कि, वे भी यह सब नही जानते हो।
Siddheshwar Shastri Chitrav [38]	सत् चा हा पसारा (विसर्ग) जेथून निर्मिला गेला, ते स्थान आकाशनिवासी जगताध्यक्षास (हिरण्यगर्भ) कदाचित ज्ञान असेल, अथवा नसेलहि.
Krishnananda [39]	Whence all creation had its origin, he, whether he fashioned it or whether he did not,he, who surveys it all from highest heaven, he knows - or maybe even he does not know.
Hari Damodar Velankar [40]	इस समूची सृष्टिका जिससे निर्माण हुआ, जिसने इसकी (बुद्धिपूर्वक) रचना की या नही, यह बात जो इस जगतका स्वामी अत्युन्नत स्वर्गमे राज्यशासन कर रहा है उसे तो सचमुच ज्ञात हो या उसे भी ज्ञात न हो।
Sāyaṇācārya [41]	उक्तप्रकारेण यथेदं जगत्सर्जनं .....

Table 3.11: Contemporary Interpretations: Verse 7 of Nāsadīya Sūkta

Analytics 3.3.7 — Proposed decoding of Verse 7. This verse fascinatingly shows correlation with the modern science. Drawing the correspondences with this verse is very critical and joyful, both!

Here, it should be noted that the first two verses (मंत्र) of Nāsadīya Sūkta (नासदीय सूक्त) discuss the situation at Pre-primordial times. Then, verses 3-4-5 will discuss the Primordial time. Now, verses 6-7 raises interesting questions and builds intuitional model of the universe.

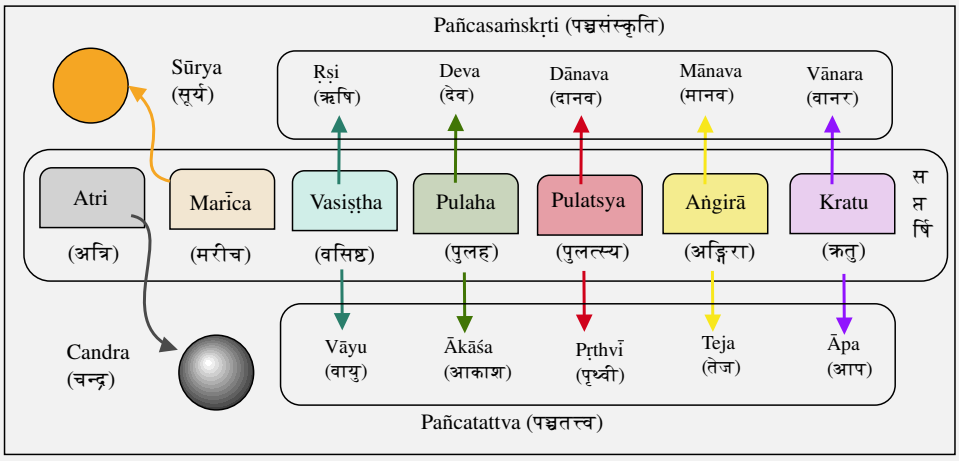
इयं विसृष्टिः यत आब भूवः- This means, This entire universe which is filled up with multiple worldly matters, it got manifested from that Supreme cause which created the effect.

यदि वा दधे यदि वा नः- That Supreme cause bears and holds this universe, or may be not!

यो अस्य अध्यक्षः परमे व्योमनः- That अध्यक्षः or the Supreme creator of this Universe, who has its abode way up in the heavens (heavenly abode) -

सो अङ्ग वेद यदि वा न वेदः- May be only that Supreme creator understand this Universe, or may be not!

- This clearly depicts the probabilistic nature of the Universe.
- The contradictory questions (यदि वा न) also suggest the finitely infinite nature of the Universe.
- In 'Mārkaṇḍeya Purāṇa' (मार्कण्डेय पुराण) the description of Bālamukunda (बालमुकुन्द) is given.
- From that seven-stage evolutionary pattern of the Universe can be mapped:
  1. Candra (चन्द्र)
  2. Pṛthvī (पृथ्वी)
  3. Sūrya (सूर्य)
  4. Brahmāṇḍa (ब्रह्माण्ड)
  5. 400 such Brahmāṇḍas (ब्रह्माण्डे) = 1 Dīrghikạ (दीर्घिका)
  6. 5000 such Dīrghikạ (दीर्घिका)
  7. The entire Universe (विश्व)
- These seven stages can also be mapped with the concept of Saptarṣi (सप्तर्षि) as follows:



- Atri gives us the moon and Mārica gives us the sun.
- Form the remaining five, we get either Pañcatattva (पञ्चतत्त्व) or Pañcasaṁskṛti (पञ्चसंस्कृति)
- For example, Pulastya is father of Viśravā and grand-father of Rāvaṇa (रावण) - thus

he gives us Dānava (दानव) lineage
- Vasiṣṭha along with Agasti was born out of 'MitrāVaruṇa' and hence gives us Vāyu.
- Aṅgirā obviously gives us Teja!
- This is how the Universe manifests!

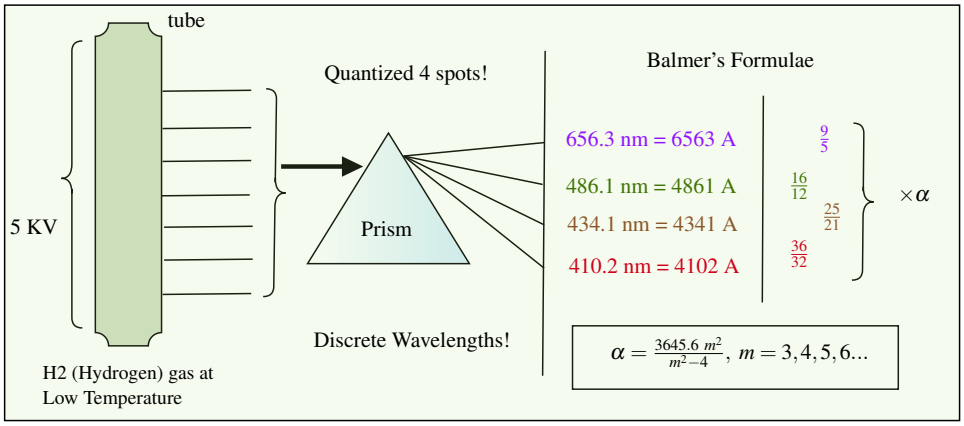
Thus Verse-7 of Nāsadīya Sūkta (नासदीय सूक्त) can be summarized as follows:

इयं विसृष्टिः यत आब भूव,	This entire universe which is filled up with multiple worldly matters; it got manifested from that Supreme cause which created the effect;
यदि वा दधे यदि वा न,	That Supreme cause bears and holds this universe; or may be not!;
यो अस्य अध्यक्षः परमे व्योमन,	That अध्यक्षः or the Supreme creator of this Universe; who has its abode way up in the heavens (heavenly abode)-;
सो अङ्ग वेद यदि वा न वेद,	May be only that Supreme creator understand this Universe; or may be not!

Drawing correspondences: Verse 7

This verse very clearly depicts the random or stochastic or probabilistic nature of the universe. This gives us the clues in the direction of realms of quantum physics.

- The Nature is governed by the rules of 'Quantum Physics'
- These rules are different than the rules of 'Classical Physics'
- E.g. Fusion inside the core of the Sun! Or conduction in solids and semiconductors.
- Even if at Macro level for certain phenomenon the classical physics works, for nano-and-micro level the Quantum physics works.
- At the Sun core, most of the phenomenon happen due to 'Hydrogen'!
- The amazing correspondence between Hydrogen and Skambha Devatā (स्कम्भ देवता) should be noted.
- Spectrum of this Hydrogen Atom is an important entry into the quantum realms.



- The Balmer's Formulae showed great intuitive understanding towards the discrete wavelengths!
- Based on these observations Bohr came up with Model of Hydrogen Atom:
  1. Electrons goes in circular orbits around photon in an atom.
  2. There are some specific orbits which do not emit radiations.

3. Angular momentum of electron about the proton is:

$$\ell = \frac{nh}{2\pi} = n\hbar \tag{3.40}$$

4. In equation (3.40), h is the Plank's constant and $n = 1, 2, 3, ...$
5. When an electron changes the orbit, the energy is emitted or absorbed in the form of EM (Electro-Magnetic) radiations.

Calculations 3.3 — Classical Physics.

- Any accelerated charged particle, when in movement, has to emit energy!
- The Potential Energy (PE) is give as:

$$PE = \frac{\ell^2}{4\pi\varepsilon_0 r} \tag{3.41}$$

- The Kinetic Energy (KE) is give as:

$$KE = \frac{1}{2}mv^2 \tag{3.42}$$

- The Force (F) is give as:

$$F = ma = \frac{mv^2}{r} \tag{3.43}$$

■

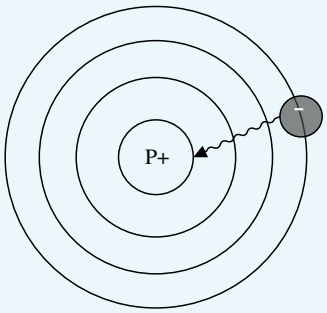
Calculations 3.4 — Quantum Physics.

- Stationary Orbits allow electrons to stay without radiating!
- This is only a boon of Quantum realms!
- For the jump of n orbits, the Energy E_n is given as:

$$E_n = -[\frac{me^4}{2 \times (4\pi\varepsilon_0)^2\hbar}] \times \frac{1}{n^2} \tag{3.44}$$

- Thus, for the jump of n orbits, the Energy E_n is given as:

$$E_n = -\frac{13.6}{n^2}\ eV \tag{3.45}$$

- 

The different Energy E values looks as follows:
$$\begin{cases} E_1, & \text{-13.6 eV;} \\ E_2, & \text{-3.4 eV;} \\ E_3, & \text{-1.5 eV;} \\ E_4, & \text{-0.85 eV;} \\ \Delta E|_{n=3\to 2}, & \text{1.9 eV;} \end{cases}$$

- The energy difference gives us the conceptualization of 'Photon'

Quantum Idea!!!

- Light has dual nature!!
- Wave form and Particle form!
- It is extremely important to note that in Antarikṣa (अन्तरिक्ष), the primordial deity (प्रधान देवता) also exhibits dual nature.
- It is Indra-Varuṇa (इन्द्र - वरुण) dual pair!
- This can just not be coincidental.
- The Indra (इन्द्र) is the particle form.
- The Varuṇa (वरुण) is of wave form.
- The wavelength is given as:

$$\lambda = \frac{hc}{\Delta E} \tag{3.46}$$

- c is the speed of light.

■

Calculations 3.5 — Limitation of Bohr's Theory.

- One of the most serious limitations is, it works only for single atom.
- For more electrons, the Columbian nature vanishes.
- Here, ions are needed to be created:

$$U(r) = -\frac{ze^2}{4\pi\varepsilon_0 r} \tag{3.47}$$

- where, z is the number of extra protons.
- The energy is given as:

$$E_n = -\frac{me^4}{2(4\pi\varepsilon_0)^2\hbar^2}\frac{z^2}{n^2} \tag{3.48}$$

■

Calculations 3.6 — Generic Formulae of Quantum Mechanics.

- The generic formulae is give as:

$$-\frac{\hbar^2}{2m}\nabla^2\psi(\vec{r}) + V(\vec{r}).\psi(\vec{r}) = E\psi(\vec{r}) \tag{3.49}$$

- The time-independent Schrödinger's Equation will take following form:

$$\nabla^2\psi = \frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2} \tag{3.50}$$

■

3.4 Asya Vāmasya Sūkta

The Asya Vāmasya Sūkta (अस्य वामस्य सूक्त) in devanāgarī (देवनागरी) with Svara (स्वर) marked is as follows:

.. Asya Vāmasya Sūkta ..

.. अस्य वामस्य सूक्त ..

1. अस्य वामस्य पलितस्य होतुस्तस्य भ्राता मध्यमो अस्त्यश्नः ।
तृतीयो भ्राता घृतपृष्ठो अस्यात्रापश्यं विश्पतिं सप्तपुत्रम् ॥ १.१६४.०१
2. सप्त युञ्जन्ति रथमेकचक्रमेको अश्वो वहति सप्तनामा ।
त्रिनाभि चक्रमजरमनर्वं यत्रेमा विश्वा भुवनाधि तस्थुः ॥ १.१६४.०२
3. इमं रथमधि ये सप्त तस्थुः सप्तचक्रं सप्त वहन्त्यश्वाः ।
सप्त स्वसारो अभि सं नवन्ते यत्र गवां निहिता सप्त नाम ॥ १.१६४.०३
4. को ददर्श प्रथमं जायमानमस्थन्वन्तं यदनस्था बिभर्ति ।
भूम्या असुरसृगात्मा क्व स्वित्को विद्वांसमुप गात्प्रष्टुमेतत् ॥ १.१६४.०४
5. पाकः पृच्छामि मनसाविजानन्देवानामेना निहिता पदानि ।
वत्से बष्कयेऽधि सप्त तन्तून्वि तत्निरे कवय ओतवा उ ॥ १.१६४.०५
6. अचिकित्वाञ्चिकितुषश्चिदत्र कवीन्पृच्छामि विद्मने न विद्वान् ।
वि यस्तस्तम्भ षळिमा रजांस्यजस्य रुपे किमपि स्विदेकम् ॥ १.१६४.०६
7. इह ब्रवीतु य ईमङ्ग वेदास्य वामस्य निहितं पदं वेः ।
शीर्ष्णः क्षीरं दुह्रते गावो अस्य वव्रिं वसाना उदकं पदापुः ॥ १.१६४.०७
8. माता पितरमृत आ बभाज धीत्यग्रे मनसा सं हि जग्मे ।
सा बीभत्सुर्गभरसा निविद्धा नमस्वन्त इदुपवाकमीयुः ॥ १.१६४.०८
9. युक्ता मातासीद्धुरि दक्षिणाया अतिष्ठद्गर्भो वृजनीष्वन्तः ।
अमीमेद्वत्सो अनु गामपश्यद्विश्वरूप्यं त्रिषु योजनेषु ॥ १.१६४.०९
10. तिस्रो मातॄस्त्रीन्पितॄन्बिभ्रदेक ऊर्ध्वस्तस्थौ नेमव ग्लापयन्ति ।
मन्त्रयते दिवो अमुष्य पृष्ठे विश्वविदं वाचमविश्वमिन्वाम् ॥ १.१६४.१०
11. द्वादशारं नहि तज्जराय ववर्ति चक्रं परि द्यामृतस्य ।
आ पुत्रा अग्ने मिथुनासो अत्र सप्त शतानि विंशतिश्च तस्थुः ॥ १.१६४.११
12. पञ्चपादं पितरं द्वादशाकृतिं दिव आहुः परे अर्धे पुरीषिणम् ।
अथेमे अन्य उपरे विचक्षणं सप्तचक्रे षळर आहुरर्पितम् ॥ १.१६४.१२

- This is 164^{th} Sūkta (सूक्त) in the first Maṇḍala (मण्डल) of Ṛgveda (ऋग्वेद).
- This hymns is exceptionally cryptic and mystic.
- This hymns has a hidden cryptographic sequence that needs to be understood to decipher this.
- This can be achieved by using a special system of interpreting numbers.
- This system is called as 'Kaṭapayādi Sūtra' (कटपयादि सूत्र).
- The crux of this system is 'अंकानां वामतो गतिः ।'.
- The following table depicts the numerical cryptic-code.

क - ट - प - य =	1
ख - ठ - फ - र =	2
ग - ड - ब - ल =	3
घ - ढ - भ - व =	4
ङ - ण - म - श =	5
च - त - - - ष =	6
छ - थ - - - स =	7
ज - द - - - ह =	8
झ - ध - - - ळ =	9
ञ - न - - - क्ष =	0 or 10

- For example the word वाम gets interpreted as 54 (म=5,वा=4)
- सप्त युजति रथ एक चक्रं । gets decoded as - सप्त=7, युजति = indicates multiplication, रथ = 72, चक्र = circle with circumference of $2 \times \pi = 6.283$, unit is 2.5. Therefore, $7 \times 72 \times 6.283 \times 2.5 = 7915$ gives average diameter of earth in miles. NASA has published it to be 7915.5 miles!
- This also enables in calculating distance between earth and sun, gravitational force of sun etc.

3.5 Devānāṃ Nu Sūkta

The Devānāṃ Nu Sūkta (देवानां नु सूक्त) in devanāgarī (देवनागरी) with Svara (स्वर) marked is as follows:

.. Devānāṃ Nu Sūkta ..

.. देवानां नु सूक्त ..

1. देवानां नु वयं जाना प्र वोचाम विपन्यया ।
   उक्थेषु शस्यमानेषु यः पश्यादुत्तरे युगे ॥ १०.०७२.०१
2. ब्रह्मणस्पतिरेता सं कर्मार इवाधमत् ।
   देवानां पूर्व्ये युगेऽसतः सदजायत ॥ १०.०७२.०२
3. देवानां युगे प्रथमेऽसतः सदजायत ।
   तदाशा अन्वजायन्त तदुत्तानपदस्परि ॥ १०.०७२.०३

4. भूर्जज्ञ उत्तानपदो भुव आशा अजायन्त ।
   अदितेर्दक्षो अजायत दक्षाद्वदितिः परि ॥ १०.०७२.०४
5. अदितिर्ह्यजनिष्ट दक्ष या दुहिता तव ।
   तां देवा अन्वजायन्त भद्रा अमृतबन्धवः ॥ १०.०७२.०५
6. यद्देवा अदः सलिले सुसंरब्धा अतिष्ठत ।
   अत्रा वो नृत्यतामिव तीव्रो रेणुरपायत ॥ १०.०७२.०६
7. यद्देवा यतयो यथा भुवनान्यपिन्वत ।
   अत्रा समुद्र आ गूळ्हमा सूर्यमजभर्तन ॥ १०.०७२.०७
8. अष्टौ पुत्रासो अदितेर्ये जातास्तन्वस्परि ।
   देवाञ्उप प्रैत्सप्तभिः परा मार्ताण्डमास्यत् ॥ १०.०७२.०८
9. सप्तभिः पुत्रैरदितिरुप प्रैत्पूर्व्यं युगम् ।
   प्रजायै मृत्यवे त्वत्पुनर्मार्ताण्डमाभरत् ॥ १०.०७२.०९

- This is also known as 'अदिति आत्मकथन' (biography of Aditi - mother of Aditya the Sun) or 'आदित्य जन्माख्यान' (genesis of Aditya the Sun).
- The third hymn of this Sūkta provides the sequence of creation of universe and it matches comprehensively with Nāsadīya Sūkta.
- This Sūkta also presents a very important concept of Uttānapada (उत्तानपद) tree.
- This also comprehensively describes the 8 Adityas (अष्टौदित्य).

3.6 Na Taa Minanti Sūkta

The Na Taa Minanti Sūkta (न ता मिनन्ति सूक्त) in devanāgarī (देवनागरी) with Svara (स्वर) marked is as follows:

.. Na Taa Minanti Sūkta ..

.. न ता मिनन्ति सूक्त ..

1. न ता मिनन्ति मायिनो न धीरा व्रता देवानां प्रथमा ध्रुवाणि ।
   न रोदसी अदुहा वेद्याभिर्न पर्वता निनमे तस्थिवांसः ॥ ३.०५६.०१
2. षड्भारञ्ऍको अचरन्बिभर्त्यृतं वर्षिष्ठमुप गाव आगुः ।
   त्रिस्रो महीरुपरास्तस्थुरत्या गुहा द्वे निहिते दर्श्येका ॥ ३.०५६.०२
3. त्रिपाजस्यो वृषभो विश्वरुप उत व्युधा पुरुध प्रजावान् ।
   व्यनीकः पत्यते माहिनावान्स रेतोधा वृषभः शश्वतीनाम् ॥ ३.०५६.०३
4. अभीक आसां पदवीरबोध्यादित्यानामह्वे चारु नाम ।
   आपश्चिदस्मा अरमन्त देवीः पृथग्व्रजन्तीः परि षीमवृञ्जन् ॥ ३.०५६.०४
5. त्री षधस्था सिन्धवस्त्रिः कवीनामुत त्रिमाता विदथेषु सम्राट् ।
   ऋतावरीर्योषणास्तिस्रो अप्यास्त्रिरा दिवो विदथे पत्यमानाः ॥ ३.०५६.०५
6. त्रिरा दिवः सवितर्वार्याणि दिवेदिव आ सुव त्रिर्नो अह्नः ।

त्रिधातु॑ राय आ सु॑वा वसू॑नि भग॑ त्रातर्धिषणे सा॒तये॑ धाः ॥ ३.०५६.०६

7. त्रिरा दि॒वः स॑वि॒ता सो॑षवीति राजा॑ना मि॒त्रावरु॑णा सुपा॒णी ।
आप॑श्चिदस्य रोद॑सी चिदु॒र्वी रत्नं॑ भिक्षन्त सवि॒तुः स॒वाय॑ ॥ ३.०५६.०७

8. त्रिरु॑त्त॒मा दू॒राशा॑ रोच॒नानि॒ त्रयो॑ राज॒न्त्यसु॑रस्य वी॒राः ।
ऋ॒तावा॑न इषि॒रा दू॒ळभा॑सस्रिरा दि॒वो वि॒दथे॑ सन्तु दे॒वाः ॥ ३.०५६.०८

- It is also known as Trisaṃkhyāyukta Kūṭasūkta (त्रिसंख्यायुक्त कूटसूक्त).
- The word 'Tri' (त्रि) is used in a cryptic way here.
- For example, in the fifth hymn, it describes Iḷā (इळा), Sarasvatī (सरस्वती) and Bhāratī (भारती).

3.7 Summary

- This chapter presented all the seven verses of Nāsadīya Sūkta (नासदीय सूक्त).
- After studying and considering all interpretations from scholars, a unique, novel, different and original interpretation of the authors is presented.
- The correspondences were drawn between these interpreted verses and the modern scientific endeavors.
- This chapter decodes Nāsadīya Sūkta systematically.
- It clearly brings out that Nāsadīya Sūkta prescribes 'Satkāryavāda'.
- Other important Sūktas are also referred and partially decoded.
- Based on this, in the next chapter we perform the experimentation with two schools of thoughts which promote Nāsadīya Sūkta, namely Sāṅkhya and Vedānta.

4. Results: Sāṅkhya and Vedānta

4.1 Evolution of Sāṅkhya Concepts

Origin of universe has been a long researched and yet an open ended problem. The problem not only brings in 'connectionism' at single layer but makes the whole framework multilayered. Every single layer is sub-layered, thus making the very nature of the problem multidimensional. The systematic scientific discoveries in the modern science and technology strangely and uniquely correlates with Vedic optics. This creates a need and desire to dig deep to bring out such highly correlated pieces [46].

One aspect of understanding the universe is through Energy mapping. It is a well proven concept in modern physics that Energy can not be created and can not be destroyed [68]. This can be related to the concept of Īśvara (ईश्वर) being attributed as Anādi (अनादि) (something that can not be created) and Anaṃta (अनंत) (something which can not be destroyed) in Vedas (वेद) and Upaniṣadas (उपनिषद्) [50]. The energy analysis takes us to deeper understanding of source of knowledge - Ṣaṭ-darśanas (षट् दर्शन) [1]. Amongst these darshanas Sāṅkhya (सांख्य) school of though by Sage Kapil (कपिल) provides profound though.

As for the origin of the universe, there has been diverse opinions amongst thinkers of different schools. While some uphold 'The theory of Creation', others maintain the 'Theory of Evolution'. Among the Creationists we have the Naiyāyikas (नैयायिक), the Buddhists - one of the Nāstikas (बौद्ध) and the Nāstikas (नास्तिक) or Nihilists. Among the Evolutionists the dominant schools are the Vedāṃtis (वेदांत) and the Sāṅkhyas (सांख्य). Even today the thought process of trying to answer the 'Origin of Universe' is very relevant [47]. Though majority of these different schools of thoughts of Indian philosophy provide metaphysical discussion, the physical forms can be logically understood to be manifested from the very metaphysical principles [51]. This gets reflected in the modern 'big-bang-theory' experimentation at physical level as depicted in Figure (4.1) [49, 69–71].

[1] Only Āstika darśanas (आस्तिक दर्शने) are considered here. Those which do not consider Veda (वेद) to be source of divine knowledge are beyond scope of this study.

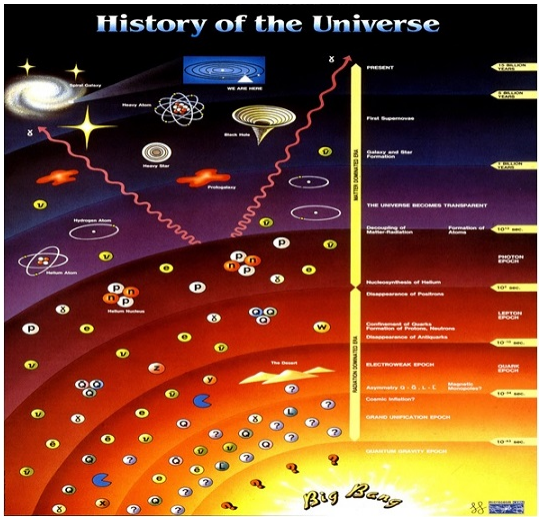


Figure 4.1: Modern 'big-bang-theory' experimentation at physical level to understand origin of universe

4.2 Sāṅkhya school of thought

Amongst different branches of Indian metaphysics [[53]]:

- A-Sat-Kārya-Vāda (अ - सत् - कार्यवाद) of Nāstikas (नास्तिक) states that a non-existent world has been produced from a non-existent cause
- And of Buddhists (बौद्ध) that an existent world has been produced from a non-existent cause
- The Abhāva-Utpatti-Vāda (अभाव - उत्पत्ति - वाद) of Naiyayikas (नैयायिक) that a non-eternal world has been produced from an eternal cause
- Vivarta-Vāda (विवर्त वाद) of Vedāntins (वेदांत) that the world is a revolution, an illusory appearance, of the one eternal reality, viz., Brahman
- Sat-Kārya-Vāda (सत् - कार्य - वाद) of the Sāṅkhyas (सांख्य) that an existent world has been produced from an existent cause.

Against the theories of A-Sat-Kārya-Vāda (अ - सत् - कार्यवाद), Abhāva-Utpatti-Vāda (अभाव - उत्पत्ति - वाद), and Vivarta-Vāda (विवर्त वाद), and in support of their theory of Sat-Kārya-Vāda (सत् - कार्य - वाद), the Sāṅkhyas (सांख्य) advance following arguments:

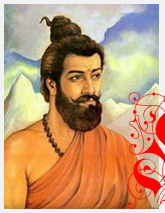
1. There can be no production of what is absolutely non-existence: e.g., a man's horn
2. There must be some determinate material cause for every product. Cream, for instance, can form on milk only, and never on water. Were it as absolutely non-existent in milk as it is in water, there would be no reason why it should form on milk, and not equally on water.
3. The relation of cause and effect is that of the producer and the produced, and the simplest conception of the cause as the producer is that it possesses the potentiality of becoming the effect, and this potentiality is nothing but the unrealized state of the effect. [2]
4. The effect is seen to possess the nature of cause, e.g., a coin still possesses the properties of the gold of which it is made.

[2]
- भेदानां परिमाणात् समन्वयात शक्तितः प्रवृत्तेश्च ।
  कारण कार्य विभागादत् अविभागादत् वैश्वरूपस्य ॥ सांख्यकारिका १५

5. Matter is indestructible; 'destruction' means disappearance into the cause.

Thus is the Doctrine of Sat-Kārya (सत् - कार्य) established. This is depicted in Kārikā 9 (कारिका ९) of Īśvarakṛṣṇa (ईश्वरकृष्ण) as shown in Figure (4.2).

The natural corollary from the above doctrine is the other doctrine of Pariṇāma-Vāda (परिणाम - वाद) or translation. It is the doctrine that, as all the effects are contained in their causes in an unmanifested form, the 'production' of the effect is nothing but its manifestation, and that, as cause and effect are essentially identical, an effect is merely a transformation or Pariṇāma (परिणाम) of the cause.



Kapila Muni (कपिल मुनि)

(Indian: Estimated to have lived in 6^{th} century BCE) was a great Hindū philosopher who is also a founder of Sāṅkhya (सांख्य) school of philosophy. He was considered as incarnation of Viṣṇu (विष्णु). Kapila, states George Williams, lived long before the composition of the Epics and the Puranas, and his name was coopted in various later composed mythologies. As per Bhāgavata Mahāpurāṇa, he was son of Kardama (कर्दम) Muni and his wife Devahuti (देवहुति).

Now, the question arises, whether the cause of the world be a single one, or whether it be manifold. Following table (4.1) summarizes various opinions in this regard [[52, 53]].

Table 4.1: Cause of universe: single or manifold

School	Proclaimer(s)	Cause	Nature of Cause
Naiyayikas (नैयायिक)	Gautama (गौतम)	Parama-Aṇus (परमाणु)	manifold (pluralism)
Vaiśeṣikas (वैशेषिक)	Kaṇāda (कणाद)	Parama-Aṇus (परमाणु)	manifold (pluralism)
Sāṅkhyas (सांख्य)	Kapila (कपिल)	Puruṣa-Prakṛti (पुरुष - प्रकृति)	Double-fold (Dualism)
Advaita (अद्वैत)	Vyaāsa (व्यास)	Cit-Tattva (चित् - तत्त्व)	Unifold (Monoism)

According to Sāṅkhyas (सांख्य), it is the Root, and is described as Puruṣa (पुरुष), that in which all things are contained, and as Prakṛti (प्रकृति) or Pradhān (प्रधान), the mother of things. The Sāṅkhyas (सांख्य) undertakes to declare and expound the successive transformations of the Pradhān (प्रधान) down to Gross Matter, with the object of accomplishing the complete isolation of the Self from even the most shadowy conjunction with the Pradhān (प्रधान). The definition of Prakṛti (प्रकृति) is that it is state of equilibrium of Sattva-Raja-Tama (सत्त्व - रज - तम), called three Guṇas (त्रिगुण). The doctrine of three Guṇas (त्रिगुण) is the very foundation of Sāṅkhyas (सांख्य) school.

At the beginning of creation, there arises in Prakṛti (प्रकृति), Spaṃdana (स्पंदन) or cosmic vibration which distributes its state of equilibrium and releases the Guṇas (गुण) from quiescence. The transformation of Prakṛti (प्रकृति) are either prakṛti-vikṛti (प्रकृति - विकृति), original or evolving as well as modifications of evolute or vikṛtī (विकृती), modifications of evolute merely [3]. How this all begins,

[3]

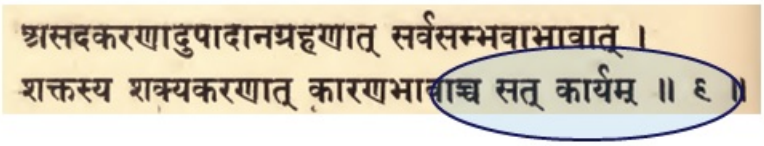


Figure 4.2: A-sat-a-karaṇāt, from the non-effectuation of the non-existent. Upādāna-grahaṇāt, from the selection of material for the effect. Sarva-sambhava-abhāvāt, from the absence of production of everything by every means. Śaktasya, of the competent. Śakya-karaṇāt, from the effectuation of the producible. Kāraṇa-bhāvāt, from the nature of the cause. Sat, existent. Kāryam, effect.

however, is the questions of importance as Sāṅkhyas (सांख्य) explains this with Paṅgu-Aṃdha Nyāya (पंगु - अंध न्याय) as depicted in Figure (4.3).

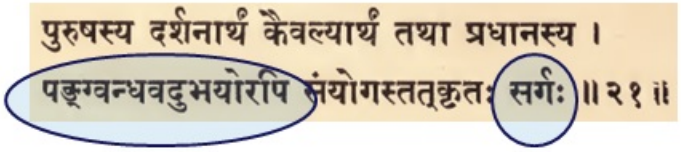


Figure 4.3: Puruṣasya, of Puruṣa. Darśana-arthaṃ, for the sake of seeing or exhibition. Kaivalya-arthaṃ, for the sake of separation. Tathā, likewise. Pradhāna-sya, of the Pradhana. Paṅgu-Andha-Vat, like that of the halt and the blind. Ubhayoḥ, of both. Api, also. Saṃyogaḥ, conjunction. Tat-kṛtaḥ, originated by that. Sargaḥ, creation or evolution.

Sargaḥ (सर्गः) means creation or evolution. This kārikā (कारिका) explains the reason for the creation of the world, however this Paṅgu-Aṃdha-Nyāya (पंगु - अंध - न्याय) needs to be looked into deeply to understand the logical reasoning as one meaning of Sāṅkhyas (सांख्य) is logic. In the next section we rigorously analyze kārikā 21 (कारिका २१) in the light of signal processing framework and try and bring out a different perspective all together [52, 53].

4.3 Signal Processing Perspective of Sāṅkhya Philosophy

In this section we bring out signal processing perspective of Sāṅkhya (साङ्ख्य) school of thought. The signal processing perspective is out and out energy perspective. Under the realms of signal processing scientists and engineers try and deal with finite energy sources. There is reason as to why signal processing perspective was chosen to analyze Sāṅkhya (सांख्य) principles and the motivation lies in the very first sloka of Ṛgveda (ऋग्वेद) [31, 44, 45, 72]. 'AUM Agnimiḷe Purohitam....' (ॐ अग्निमिळे पुरोहितम् ..) the very first śloka (श्लोक) hypothecates the availability of free energy in space. This provides solid reasoning to probe the Sāṅkhya (सांख्य) logic to energy analysis. The coincidental part is the very first kārikā (कारिका) by Īśvarakṛṣṇa (ईश्वरकृष्ण) also presents an existence of Coherent, Perpetual, dynamic and unmanifested space. This is shown in Figure (4.4).

- मूल प्रकृतिर् अविकृतिर् महद् अद्याः प्रकृति विकृतयः सप्त ।
षोडशकस्तु विकारो न प्रकृतिर्न विकृतिः पुरुषः ॥ सांख्यकारिका ३

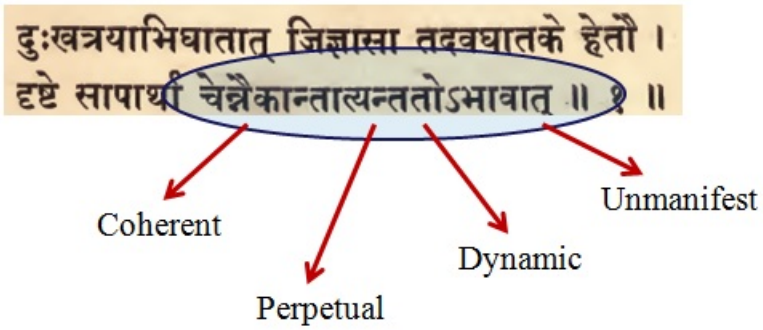


Figure 4.4: Duḥkha-traya-abhighātāt, from the disagreeable occurrence affection or action of the threefold pain or cause of suffering. Jijñāsā, the desire to know, enquiry. Tat-avaghātake, prevention or counter-active thereof. Hetau, into the means. Dṛṣṭe, there existing visible or ordinary means. Sā, the enquiry. Apa-arthā, purposeless, superfluous. Cet, if it is said. Na, no. Ekānta-atyanta-ataḥ-Abhāvāt, Coherent, Perpetual, dynamic and unmanifest

4.3.1 Sarga as per Sāṅkhya

As per Sāṅkhya (सांख्य) school of thought, the objective world contains twenty-four Tattvas [4], namely:

- Prakṛti (प्रकृति)
- Mahat (महत्)
- Ahaṃkāra (अहंकार)
- Manas (मन)
- Five iṃdriyas of cognition (ज्ञानेंद्रिय)
- Five iṃdriyas of Action (कर्मेंद्रिय)
- Five tana-mātrās (तन्मात्रा)
- Five gross elements (पंच महाभूते)

The Puruṣa (पुरुष) is isolated 25^{th} Tattva which does not take part in the evolution process, however is Sākśī (साक्षी) to the entire process [5]. When cosmic vibration disturbs equilibrium state of Prakṛti (प्रकृति), Rājas (राजस) at once acts upon Sattva (सत्त्व) and manifests it as Mahat (महत्), wherein Mahat (महत्) denoted Buddhī (बुद्धी). From Buddhī (बुद्धी) springs Ahaṃkāra (अहंकार) which is literally the I-maker. The modification of Ahaṃkāra (अहंकार) is twofold, as it is influenced by Sattva (सत्त्व) or by Tāmasa (तामस). The Sāttvika (सात्त्विक) modifications are the eleven Indriyas, that is, the five Indriyas of Cognition (ज्ञानेंद्रिय), viz. the powers located in Eye, Ear, Nose, Tongue and Skin, the five Indriyas of Action (कर्मेंद्रिय), viz. the powers located in voice, hand, feet, organs of generation and organs of excretion. The eleventh is Manas (मन), which is both power of cognition as well as action. The Tāmasika (तामसिक) modifications of Ahaṃkāra (अहंकार) are the five Tana-mātrās (तन्मात्रा), viz. of Sound, Touch, Form, Flavour and Smell. They are pure, subtle or simple elements, the metaphysical parts of the ordinary atoms of matter. The transformations of the Tana-mātrās

[4]
- प्रकृतेर्महांस्ततोऽहङ्कारस्तस्माद् गणश्च षोडशकः ।
तस्मादपि षोडशकात् पञ्चभ्यः पञ्च भूतानि ॥ सांख्यकारिका २२

[5]
- सिद्धं साक्षित्त्वमस्य पुरुषस्य ॥ सांख्यकारिका १९

(तन्मात्रा) are the Gross Elements of Ether, Air, Fire, Water and Earth - the ordinary atoms of matter. This entire process is captured in Figure (4.5).

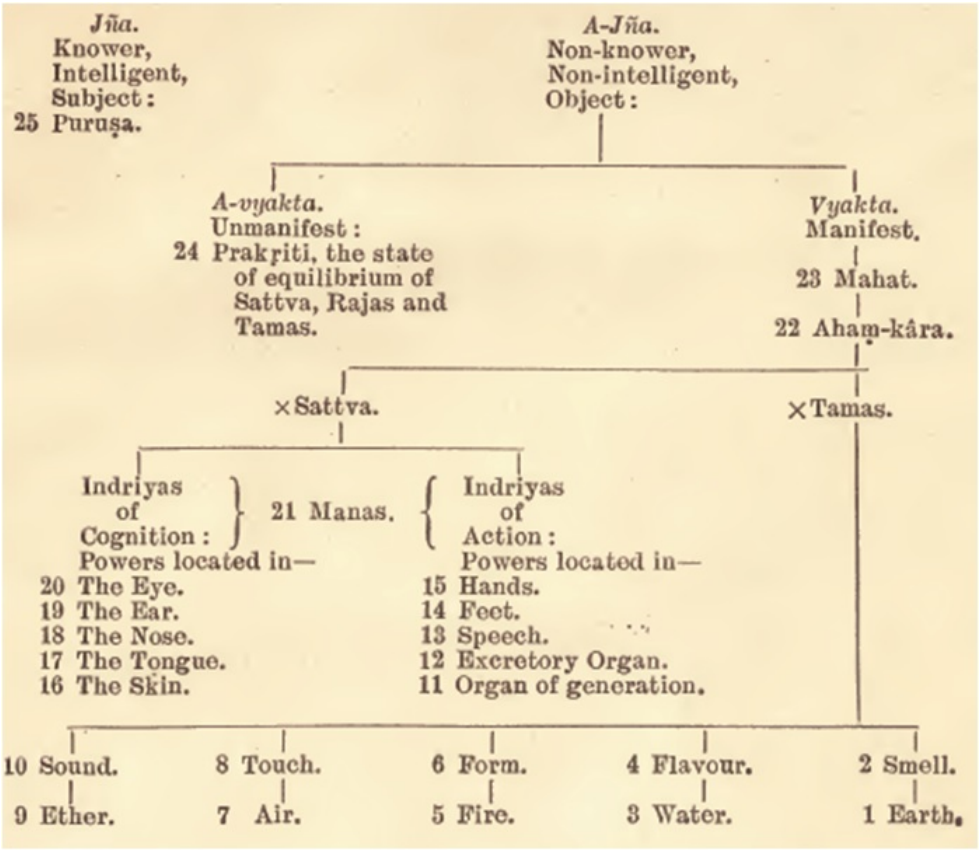


Figure 4.5: Sarga - Evolutionary process as per Sāṅkhya.

Throughout this process, Prakṛti (प्रकृति) is the one that undergoes transformations, however bring Jaḍa (जड) it won't be able to undergo transformations on it's own. Puruṣa (पुरुष) being Cetana (चेतन) provides the necessary Adhiṣṭhāna (अधिष्ठान), however remains totally isolated from the process of evolution. This union of Puruṣa (पुरुष) and Prakṛti (प्रकृति) is explained by Paṅgu-Aṃdha Nyāya (पंगु - अंध न्याय) by Sāṅkhya (सांख्य) [52, 53].

4.3.2 Signal Processing Perspective of Sarga

Puruṣa (पुरुष) is the principle of 'being' and Prakṛti (प्रकृति) is the principle of 'becoming', their union for the evolution to happen is hard to understand. Puruṣa (पुरुष) eternally is and never becomes, while Prakṛti (प्रकृति) is essentially movement. We analyze this using energy analysis in Hilbert domain.

Hilbert space is an infinite dimensional, complex vector space and comprises of an infinite set of ortho-normal basis vectors with complex coefficients. The infinite size of the space basically indicates that any physical system can characterize the behavior in infinite number of physically inequivalent states. This abstract, quantum-mechanical space of all possibilities, called as Hilbert space, is good starting point to visualize and map Coherent, Perpetual, dynamic and unmanifested form of the space. We will show that this hypothesis holds good through implementation and equivalence of 'Puruṣa-Prakṛti' (पुरुष - प्रकृति) with 'Scaling function-Wavelet Function' from Wavelet theory [48, 73, 74].

The modified evolutionary process is captured in Figure (4.6).

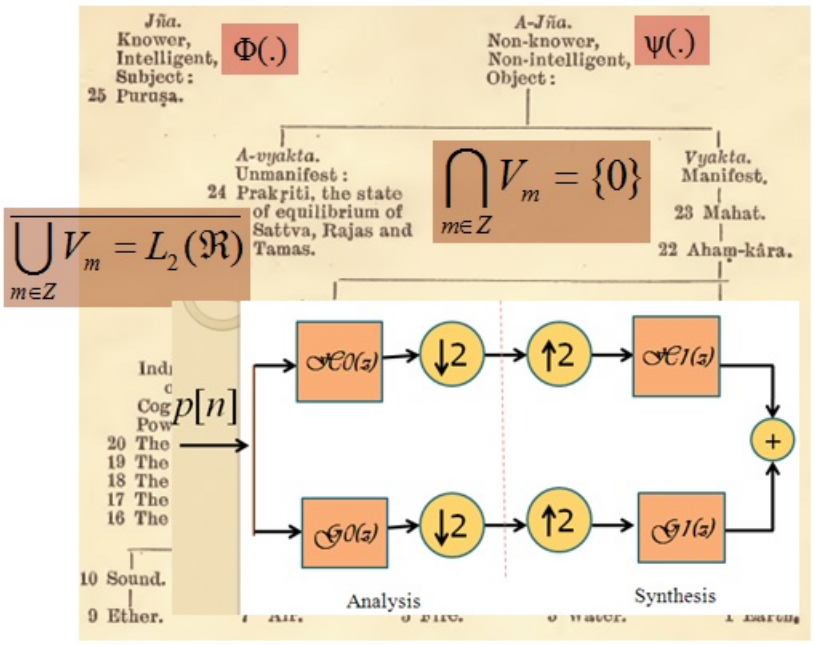


Figure 4.6: Modified Sarga - Evolutionary process as per Sāṅkhya from Wavelets perspective.

Conventionally wavelet transform decomposes signal into two separate series. One is a single series which represents coarse version which leads to scaling function, popularly known as the father function. Other is a double series which represents a refined version which leads to wavelet function, popularly known as mother wavelet. Father function or scaling function typically represented as $\phi(.)$ can be thought of to be similar to Puruṣa (पुरुष) from the Sāṅkhya (सांख्य) concepts and mother wavelet typically represented as $\psi(.)$ can be thought of to be similar to Prakṛti (प्रकृति) from Sāṅkhya (सांख्य) concepts. They are similar in following ways:

- Just like Puruṣa (पुरुष) and Prakṛti (प्रकृति) together produce Sarga (सर्ग) [6], ϕ and ψ functions together produce entire family of wavelet.
- As Puruṣa (पुरुष) is Taṭastha (तटस्थ) in evolution process, ϕ function does not get involved in the actual analysis.
- The way Prakṛti (प्रकृति) brings out evolution, mother wavelet function ψ produces ladder of subspaces thus producing multi-resolution effect.
- Prakṛti (प्रकृति), however, won't be able to do evolution alone, similarly ϕ and ψ produce projection in V_j and W_j subspaces thus creating nested subspaces.
- Sāṅkhya (सांख्य) proclaims the plurality of Puruṣa (पुरुष), similarly there are multiple wavelet families.
- Prakṛti (प्रकृति) is blind without Puruṣa (पुरुष), from the dilation equation we can conclude the same that ψ gets blinded without ϕ.

6
– संयोगस्तत्कृतः सर्गः ॥ सांख्यकारिका ॥२१॥

4.4 Evolution of Vedānta Concepts

- In this section we build on top of the Sāṅkhya concepts from the previous sections. We bring in the signal processing perspective to uncover the Advait Vedānta (अद्वैत वेदान्त) philosophy.
- It will also be demonstrated that eventually Advait Vedānta (अद्वैत वेदान्त) overrules Sāṅkhya concepts and monoism outperforms dualism.
- This is shown crisply as lower resolution ψ emerges from higher resolution ϕ. Thus, towards the completion of the L_2 norm, everything adds up to that ONE as proclaimed by Advait Vedānta (अद्वैत वेदान्त).
- Although both the Sāṅkhya as well as Vedānta (अद्वैत वेदान्त) doctrines support the concept of Satkāryavāda (सत्कार्यवाद), they differ in the basic conceptualization of the Nimitta (निमित्त) and Upādāna (उपादान) Kāraṇa (reason) behind the creation of the universe.
- As per Advait Vedānta (अद्वैत वेदान्त), the Nimitta (निमित्त) and Upādāna (उपादान) Kāraṇa (reason) behind the creation of the universe is one and the same.
- The lineage that we trace for comprehensive analysis of Advait Vedānta Philosophy is as follows: Bhagavāna Nārāyaṇa (भगवान नारायण) - Brahmadeva (ब्रह्मदेव) - Vasiṣṭha (वसिष्ठ) - Śakti (शक्ति) - Parāśara (पराशर) - Vyāsa (व्यास) - Śuka (शुक) - ..... - Gauḍapāda (गौडपाद) - Govindapāda (गोविन्दपाद) - Śaṅkarācārya (शङ्कराचार्य) - ..... [7]
- The reference for Advait Vedānta Philosophy includes Prasthānatrayī (प्रस्थानत्रयी) as follows:
  1. Brahmasūtras (ब्रह्मसूत्रे) + Śāṅkarabhāṣya (शाङ्करभाष्य)
  2. Upaniṣadas (उपनिषदे)
  3. Śrīmadbhagavadgītā (श्रीमद्भगवद्गीता)

Analytics 4.4.1 — Fundamentals of Advaita. The crux of the Advaita Vedānta (अद्वैत वेदान्त) is captured in the first 4 Sūtras (सूत्रे) of Uttara Mīmāṃsā (उत्तर मीमांसा) or BrahmaSūtras (ब्रह्मसूत्रे):

1. अथाऽतो ब्रह्मजिज्ञासा।
2. जन्माद्यस्य यतः।
3. शास्त्रयोनित्वात्।
4. तत्तु समन्वयात्।

- From these it is clear that Parabrahma (परब्रह्म) is that ONE as per Advait Vedānta (अद्वैत वेदान्त).
- From the framework that we have developed, to prove this, what we need to show is as follows:
  1. The Father function or the Scaling equation or the Low pass filter or $\phi(.)$ is the Śapala Brahma (शपल ब्रह्म).
  2. The Mother function or the Wavelet equation or the High pass filter or $\psi(.)$ is the Māyā (माया).
  3. We need to show that $\phi(.)$ in the finer resolution can produce the $\phi(.)$ and $\psi(.)$ in the coarser resolution.

[7]
– ॐ नारायणं पद्मभवं वसिष्ठं शक्तिं च तत्पुत्रपराशरं च ।
व्यासं शुकं गौडपदं महान्तं गोविन्दयोहीन्द्रमथास्य शिष्यम् ।
श्रीशङ्कराचार्यमथास्य पद्मपादं च हस्तामलकं च शिष्यं ।
तं तोटकं वार्तिककारमन्यानस्मद्गुरुं संततमानतोऽस्मि ॥ गुरुस्तोत्रम् २०-२१

4. This $\phi(.)$ as we reach out to the finest resolution gives us L_2 norm.
5. This L_2 norm with the closure is nothing else but the Parabrahma (परब्रह्म).
6. All the $\phi(.)$ and $\psi(.)$ pairs in the resolutions lesser than the L_2 norm are the corresponding Śapala Brahma (शपल ब्रह्म) and Māyā (माया) pair in the corresponding resolution and scale.

- We now prove this in this chapter using the signal processing framework.
- It must however be noted that Advait Vedānta philosophy (अद्वैत वेदान्त तत्त्वज्ञान) is the paramount of Indian Philosophy and one needs to get equipped to understand the subtle details. This could be the reason why Lord Kṛṣṇa mentions 'कश्चिन्मां वेत्ति तत्त्वतः ।' in Bhagavadgītā (भगवद्गीता).
- The four pre-requisites have been provided by Sadānanda (सदानंद).[a]
- The four pre-requisites or 'साधनचतुष्टय' is as follows:
  1. नित्यानित्यवस्तुविवेक - Prudent confirmation that 'Brahma' ($\phi(.)$) is the only सद्वस्तू or real operator.
  2. इहामुत्रार्थफलभोगविराग - Detachment from the physical materialistic pleasures.
  3. शमादिष्टकसम्पद - Śama (शम) = serenity of mind, Dama (दम) = control of senses, Uparati (उपरति) = satiety, Titikṣā (तितिक्षा) = endurance, Śraddhā (श्रद्धा) = unflinching devotion, Samādhāna (समाधान) = content and satisfaction.
  4. मुमुक्षुत्व - Intensified desire for enlightenment.

[a] In Vedāntasāra (वेदान्तसार), Sadānanda (सदानंद) proclaims - 'अधिकारी तु विधिवदधीतवेदवेदाङ्गत्वेन आपाततः अधिगताखिलवेदार्थः अस्मिन् जन्मनि जन्मान्तरे वा काम्यनिषिद्धवर्जनपुरःसरं नित्यनैमित्तिकप्रायश्चित्तोपासनानुष्ठानेन निर्गतनिखिलकल्मषतया नितान्तनिर्मलस्वान्तः साधनचतुष्टयसम्पन्नः प्रमाता ।'

Śaṅkarācārya Ādi (आदि शङ्कराचार्य)

(Indian: Early 8^{th} century Indian Philosopher) was the greatest of the Hindū philosophers who reinvented Advaita Vedānta (अद्वैत वेदान्त) school of philosophy. He exemplified confluence of Parā-Bhakti (परा - भक्ति) and Parā-Jñāna (परा - ज्ञान). Along with his scholarly works like Śārīraka Bhāṣya (शारीरक भष्य) and Vivekacuḍāmaṇi (विवेकचुडामणि), he has also written beautiful Stotras with great exhibition of devotion.

4.5 Relation Between ϕ, ψ and Filters: Towards understanding Māyā

In this section we will demonstrate how to relate $\psi(t)$ and $\phi(t)$ of the MRA to the filter bank by studying the generic structure of the analysis and synthesis filter banks. We have already drawn the correspondence that $\phi(.)$ is the Śapala Brahma (शपल ब्रह्म) and $\psi(.)$ is the Māyā (माया). It must have been obvious by now that such a connection exists. In fact we will soon demonstrate that ψ or माया

emerges out if ϕ or ब्रह्म.

After the theoretical part (औपपत्यात्मक), we will also build the filter bank out of the idea of multi-resolution analysis with the Haar MRA as an example (प्रयोगात्मक). So before proceeding any further let us make a few generalizations which will help us to build that relationship more comprehensively. We will arrive at the general structure of the analysis and the synthesis filter banks based on the study of the Haar MRA. The complete characterization of these functions emerges out of the preface (उपोद्घात) written by Ādi Śaṅkarācārya (आदि शङ्कराचार्य).

The Jñāna Adhyāsa (ज्ञान अध्यास) is given by the first two sentences:

1, युष्मत् - अस्मत् - प्रत्ययगोचरयोः - विषय - विषयिणोः - तमः प्रकाशवत् विरुद्ध स्वभावयोः - इतरेतर - भाव - अनुपपत्तौ - सिद्धायाम् ।

2, तद्धर्माणामपि - सुतराम् - इतरेतर - भावानुपपत्तिः - इत्यतः ।

Similarly, Artha Adhyāsa (अर्थ अध्यास) is given by the next three sentences:

3, अस्मत्प्रत्ययगोचरे - विषयिणी - चिदात्मके - युष्मत्प्रत्ययगोचरस्य - विषयस्य तद्धर्माणां च - अध्यासः ।

4, तद्विपर्ययेण विषयिणः - तद्धर्माणां च विषये - अध्यासो ।

5, मिथ्येति भवितुं युक्तम् ।

These statements are very crisp and using them the following can be clearly stated:

- Here अस्मत् or अहं clearly indicates the Viṣayī (विषयी) or Ātmā (soul) or Brahma (ब्रह्म) the Supreme Soul.
- युष्मद् or इदं indicates the Viṣaya (विषय) or the objects of sensory organs or Māyā (माया).
- They are opposites of each other and therefore the '.' or 'dot' or 'inner' product between ϕ and ψ is zero indicating the orthogonality.
- As already proved in the third chapter, this opposite nature is of the form of Dvaita (द्वैत) and not Dvandva (द्वन्द्व).
- The manifestation of the world happens through Adhyāsa (अध्यास).
- The nature of it is 'Satyānṛte Mithunīkṛtya' (सत्यानृते मिथुनीकृत्य).
- This is captured as MRA or Multi-resolution Analysis framework.

4.5.1 Haar MRA - Decoding Sarga and Visarga

In this section, we will try to establish a relation between the functions $\phi(t)$ (शपल ब्रह्म), $\psi(t)$ (माया) and the filter banks. Let us focus our attention on Haar MRA to explore more on understanding Sarga (सर्ग) and Visarga (विसर्ग) of Sṛṣṭī (सृष्टी). We use Dyadic Multi-resolution Haar analysis to prove the concepts.

Relation between the Śapala Brahma function $\phi(t)$ and filter banks

For Haar MRA, $\phi(t)$ is the basis of V_0 *i.e* $\phi(t) \in V_0$. Since this function is confined to finite linear sub-set, it captures the essence of being the Śapala Brahma function. Also recall that there exists a ladder of spaces in MRA which states that $V_0 \subset V_1$. This leads to the conceptualization of nested sub-spaces, which has been vividly expressed in Advaita Vedānta (अद्वैत वेदान्त) philosophy [8].

$\phi(t)$ should therefore be expressible in the basis of V_1 *i.e.* $\phi(2t-n)$, $n \in \mathbb{Z}$.

Analytics 4.5.1 — Connecting $\phi(t)$ and filter banks. From the figure we see that $\phi(t)$ is expressible in its own dilates and translates. This dilation property is captured as अत्ता चराचरग्रहणात्

[8] e.g. यद्यत्सुखं भवेत्तत्तद्ब्रह्मैव प्रतिबिम्बनात् । पञ्च. १५ -१९., here प्रतिबिम्ब refers to the nested sub-sets which leads to Multi-Resolution Analysis

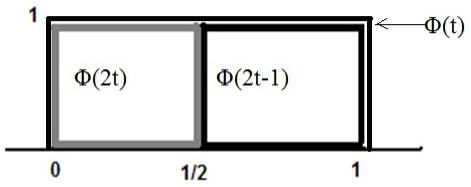


Figure 4.7: Expressing $\phi(t)$ in term of dilates and translates

। वेदान्तसूत्र. ०९.

$$\phi(t) = \phi(2t) + \phi(2t-1)$$

The above equation is called as the **dilation equation**. If we write down the coefficients of $\phi(2t-n)$ as a sequence, we get the impulse responses corresponding to the Low-pass filter.

$$\begin{matrix} 1 & 1 \\ \uparrow & \\ 0 & \end{matrix}$$

This is in compliance with भेदव्यपदेशाच्चान्यः । वेदान्तसूत्र. २१, which in turn follows Chāndogya Upaniṣada [a].
The dilation equation can thus be modified in a generalized way as follows:
If $h[n]$ is the impulse response of the low pass filter in the two band filter bank then,

$$\phi(t) = \sum_{n\in\mathbb{Z}} h[n]\phi(2t-n)$$

[a] अथ य एषोऽन्तरादित्ये हिरण्मयः पुरुषः । छां. १-६-६

Relation between the Māyā function $\psi(t)$ and filter banks

$\psi(t) \in V_0$ should also be expressible in terms of basis of V_1 *i.e.* $\phi(2t-n)$, $n \in \mathbb{Z}$.

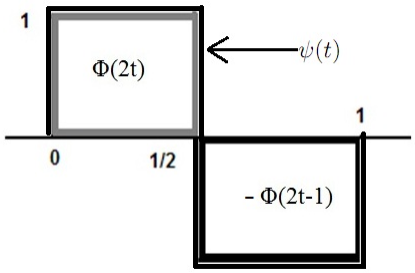


Figure 4.8: Expressing $\psi(t)$ in term of dilates and translates

Analytics 4.5.2 — Connecting $\psi(t)$ and filter banks. Graphically, from the figure 4.8, we can see that the dilation equation will be of the form

$$\psi(t) = \phi(2t) - \phi(2t-1)$$

If we write down the coefficients of $\phi(2t-n)$ as a sequence, we get the impulse responses corresponding to the High-pass filter.

The dilation equation can thus be modified in a generalized way. A doubt may arise here as to whether this is in line with Vedas? Indeed, it is. We confirm this through रूपं रूपं प्रतिरूपो बभूव तदस्य रूपं प्रतिचक्षणाय । इन्द्रो मायाभिः पुरुरूप ईयते युक्ता ह्यस्य हरयः शता दशेति ॥ (बृ. २-५-१९, ऋ. ६-४७-१८). If $g[n]$ is the impulse response of the high pass filter in the two band filter bank then,

$$\psi(t) = \sum_{n\in\mathbb{Z}} g[n]\phi(2t-n)$$

This implies that, if we know the impulse responses, we can go the reverse way round to generate $\phi(t)$ and $\psi(t)$.
Thus in short,
Low-pass filter $\rightarrow$ scaling function expansion (Brahma or ब्रह्म function)
High-pass filter $\rightarrow$ wavelet expansion (Māyā or माया function)
Therefore, we can completely characterize the system $\phi(t)$ and $\psi(t)$ if we know their dilation equation.

1 - 1
↑
0

Now, the concept of Parabrahma (परब्रह्म) being the L_2 norm can be established only if we prove that the filter bank structure that emerges out is perfectly reconstructible and there are no energy losses. Before doing that we will quickly visit the implementation strategy for this in the form of Two-Band Filter Bank.

4.6 Two Channel Filter Bank

This structure is often used in the implementation of discrete wavelet transform. In this structure we ensure orthogonality by relating the Z-transform at every point of following structure and finding relationship between $H_0(Z)$, $H_1(Z)$, $G_0(Z)$ and $G_1(Z)$ which ensures perfect reconstruction at $Y(Z)$. We also aim to enhance our analysis to understand 'conjugate quadrature filter banks'.[9]

The two channel filter bank is shown in figure 4.9.

[9]The conjugate quadrature nature is extremely critical. Here once again we refer to the preface or उपोद्घत of शारीरकभाष्य of आदि शङ्कराचार्य as - तथापि अन्योन्यास्मिन अन्योन्यात्मकताम् अन्योन्यधर्मान च अध्यस्य इतरेतर अविवेकेन ... ।. The word अन्योन्य indicates the mutual part of it. The mutual nature also protects the energy loss and makes the complete structure complementary.

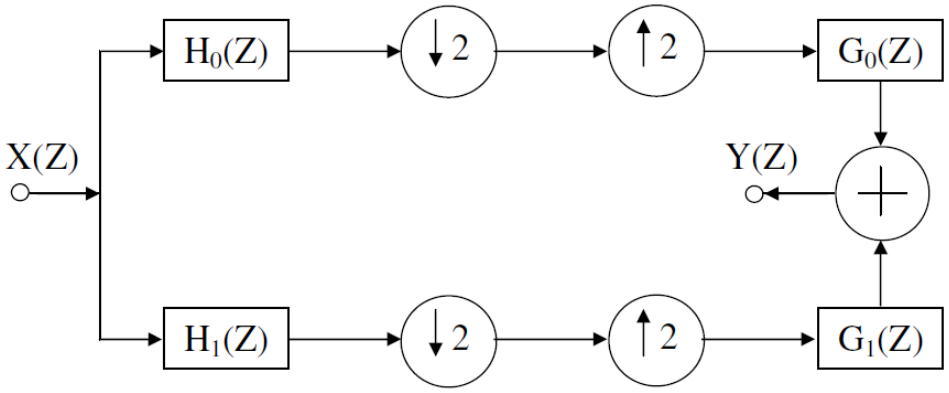


Figure 4.9: Two Channel Filter Bank

4.7 Haar Wavelet

To explain the Multi-Rate framework and then initially fit Sāṅkhya (सांख्य) principles and then elevate it to Vedānta (वेदान्त) principles accordingly, we introduce the Haar Wavelet construct here first.

In this section we shall get introduced to the Haar Multi-resolution Analysis. Haar was a mathematician, and he gave a very radical idea that any continuous function can be represented in the form of discontinuous functions, and by doing so one can go to any level of continuity that one desires. This is the central idea in the 'Haar way' of representing functions. This exactly gets captured in the Taṭastha Lakṣaṇa (तटस्थ लक्षण) of Brahma (ब्रह्म) [10]. We start from a very discontinuous function and make it smoother and smoother by adding more and more discontinuous functions (which is in a way some additional information) to it until we reach arbitrarily closer to the continuous function that we are trying to approximate. The same concept is being captured by Śrībādarāyaṇācārya (श्रीबादरायणाचार्य) in Vedānta Sūtras (वे.सू. १.२.५.१८) This idea is opposite to the idea of the Fourier transform. One can relate that in Fourier transform the discontinuous function is represented in the form of smooth continuous function.

lfred Haar

(Hungarian: *Haar Alfréd*; 11 October 1885, Budapest 16 March 1933, Szeged) was a Hungarian mathematician. In 1904 he began to study at the University of *Göttingen*. His doctorate was supervised by David Hilbert. The Haar measure, Haar wavelet, and Haar transform are named in his honor. The Haar sequence is now recognized as the first known wavelet basis and extensively used as a teaching example.

[10] जन्माद्यस्य यतः । ब्रह्मसूत्र-१.१.२, which is also being supported with a Śrutī (श्रुती) as यतो वा इमानि भूतानि जायन्ते, येन जातानि जीवन्ति, यत्प्रयन्त्यभिसंविशन्ति, तद्विजिज्ञासस्व तद् ब्रह्म । तै.उ.३.१

4.8 The Haar wavelet

We use Haar wavelet to illustrate the entire construct. The function shown in the figure 4.10 is the Haar Wavelet or Haar Māyā (माया) function.

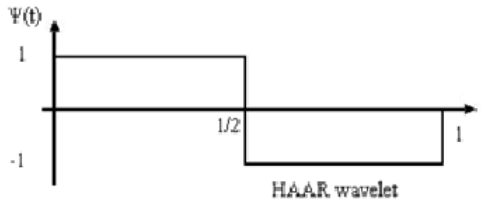


Figure 4.10: $f_2(t) - f_1(t)$ Haar Wavelet or Māyā function

This function is represented as $\psi(t)$. By using scalar multiplication and delaying, we can see that $f_2(t) - f_1(t)$ can be reconstructed from $\psi(t)$. Thus,

$$f_2(t) - f_1(t) \;=\; -h_1 \times \psi(t/T) + h_2 \times \psi(\frac{t-T}{T})$$

here h_1 and h_2 have usual connotation. The function $\psi(t)$ is called the **Haar wavelet or Māyā function**. In general when we start with $\psi(t)$ we can construct a function $\psi(\frac{t-\tau}{s})$ as a building block, where *'s'* is positive real and τ should be real. This dilation and translation ability of Wavelet function is responsible for the effective projection of any signal. In Brahmasūtrabhāṣya (ब्रह्मसूत्रभाष्य) 1-3-19, it clearly states: एक एव परमेश्वरः कूटस्थनित्यो विज्ञानधातुः अविद्यया मायया मयाविवत् अनेकधा विभाव्यते ।

The variable *'s'* dilates $\psi(t)$ and *'*τ*'* translates $\psi(t)$. The variable τ is called the **translation index** and the variable *'s'* is called the **dilation variable**. If we consider time intervals of length *T/2* for piecewise constant approximation then the value of *'s'* is *T/2*, if length of time interval is T then s=T. It means the single function $\psi(t)$ allows you to bring in resolution step by step to any level of detail. Thus, by dividing T into smaller subdivisions of *T/2, T/4* and so on, any function $x_a(t)$ can be made **arbitrarily close** to original function $x(t)$. This ability is clearly stated in Śvetāśvetara Upaniṣad (श्वेताश्वेतर उपनिषद्) 4-10: मायां तु प्रकृति विद्यान्मायिनं तु महेश्वरम् । This also brings in important point that in connotation, Māyā and Prakṛti are similar.

If $x_e(t)$ denotes the error due to approximation, it can be expressed as

$$\begin{aligned} x_e(t) &= x(t) - x_a(t) \\ \zeta &= \int_{-\infty}^{\infty} |\, x_e(t)\,|^2\, dt \end{aligned}$$

Where, ζ is the squared error. What we mean by arbitrarily close is that for any fixed value of $\zeta(>0)$, we can always find a positive integer **m** such that a piecewise constant approximation of $x(t)$ with an interval of $T/2^m$ satisfies the requirement of ζ.
It must be noted that any signal can be represented in piece wise constant form **if and only if** it has **finite energy**. This once again confirms our discussion on Verse 1-3-5 of Nāsadīya Sūkta (नासदीय सूक्त). The analysis that we demonstrate is Energy analysis.

4.9 L_k norms of $x(t)$

Calculations 4.1 — From infinite (∞) to finite. A function has finite energy content implies and is implied by its L_2 norm being finite. So what is this L_2 norm? The L_2 norm of a signal is defined as

$$L_2 \; norm \; of \; x(t) \; = \left[\int_{-\infty}^{\infty} | \, x(t) \, |^2 \, dt \right]^{\frac{1}{2}}$$

A doubt may arise here:

- With reference to Svarūpalakṣaṇa (स्वरूपलक्षण) of Brahma (ब्रह्म), it is in infinitely blissful state and hence captured as 'सत्यं ज्ञानमनन्तं ब्रह्म ।' in (2-1) of Taittirīya Upaniṣada (तैत्तिरीय उपनिषद).
- As per Vedānta philosophy, Brahma is also Upādāna Kāraṇa of the world. (Bra.Sū.1-1-3: शास्त्रयोनित्वात ।).
- TaittirīyaBhāṣya clearly states: प्रागभावप्रध्वंसाभावाप्रतियोगित्वं कालतः आनन्त्यम् ।
- If Brahma is limitless and beyond the realms of time, space and dimension, why are we considering L_2 norm for finite energy signal here?

This can be answered systematically as follows:

- That reflection of Brahma which controls Māyā is called as Īśvara (ईश्वर).
- The limited form of the limitless is termed as Śapala (शपल) or Śabala (शबल) brahma.
- This limitedness is also expressed by specific terms in Pañcadaśī (पञ्चदशी).
- As per Pañcadaśī (पञ्चदशी) the journey from infinite to finite is through stages including: Mahākāśa (महाकाश) corresponding to Brahma (ब्रह्म) to Abhrākāśa (अभ्राकाश) corresponding to Īśvara (ईश्वर). Similarly, from Ghaṭākāśa (घटाकाश) corresponding to Kūṭastha (कूटस्थ) to Jalākāśa (जलाकाश) which corresponds to Jīva (जीव).
- Therefore, we work with the limited form of 'Brahma' and hence work out the L_2 norm for energy analysis.

In general, we can define the L_p norm of x(t) as

$$L_p \; norm \; of \; x(t) \; = \left[\int_{-\infty}^{\infty} | \, x(t) \, |^p \, dt \right]^{\frac{1}{p}}$$

where p is any real number. The L_∞ norm of $x(t)$ is defined as

$$L_\infty \; norm \; of \; x(t) \; = \lim_{p \to \infty} \left[\int_{-\infty}^{\infty} | \, x(t) \, |^p \, dt \right]^{\frac{1}{p}}$$

■

Significance of L_∞ norm

As the value of p increases, large values in $x(t)$ are being emphasized. This happens because for a large p, the integral will have a large contribution from higher values in $x(t)$.

$L_2(\mathbb{R})$ is said the to be the space of all **real** functions whose L_2 norm is finite. The word 'space' is used with the intent that if we take a linear combination of two or more functions in that set then we get back a function in that set. As we reduce the size of the individual time intervals previously considered, we said we could as close to the original function as we desire by reducing the error. This clearly means that L_2 norm of the error can be reduced to as small a value as we desire. The

Fourier series allows us to do this for a reasonable class of functions.

The same kind of thing is happening here. Just on function $\psi(t)$ is able to take us as close as we desire to the function which we wish to approximate. And this is just one $\psi(t)$. The whole subject of wavelets allows us to build many such $\psi(t)$'s. Each time we improved the resolution by factors of '2' and hence the term 'dyadic' which represents steps of two was used in the beginning of the chapter. Thus the Haar wavelet is an example of a dyadic wavelet. In the following chapters we will mostly focus our attention on dyadic wavelets. Dyadic wavelets are the most easily designed, the best and most easily implemented and also the best understood.

If one understands the 'Haar wavelet' and if one understands the way in which the Haar MRA is constructed many concepts of multi-resolution analysis would become clear. In this chapter we brought out the idea of the Haar wavelet explicitly. We now know that dilates and translates of this function can capture information in going from one resolution to the next level of resolution in steps of two each time. So in terms of the spaces we are actually going from one subspace of $L_2(\mathbb{R})$ to the next subspace.

This whole concept of building nested sub-sets leading to formation of the ladder of sub-spaces in extremely important. This is because as we move up the ladder we achieve L_2 norm leading to the conceptualization of Śapala Brahma (शपल ब्रह्म) and down the ladder gives us the trivial space $\{0\}$ leading to Jīvātmā (जीवात्मा). For explaining the notional constructs behind doctrines like 'Tat-Tvam-Asi' (तत् - त्वम् - असि), we can use Jahadajahallakṣaṇā (जहदजहल्लक्षणा) only due to the ladder of nested sub-sets [11]. Here 'Tat' refers to Śapala Brahma or Īśvara and 'Tvam' refers to Jīvātmā. As per Jahadajahallakṣaṇā, we need to give up the limitedness of Jīvātmā and unlimitedness (perfect norm reconstruction) of Īśvara, which is possible by moving from one resolution and/or scale to another!

The next task is to see how the dilates and translates of Haar wavelet help us in adding more and more to the subspaces in going from a coarser subspace all the way to $L_2(\mathbb{R})$ on one side and all the way down to a trivial subspace on the other (also referred to as the ladder of subspaces). Further we will bring out the idea of the basis of these subspaces and how the Haar wavelet helps in capturing the difference subspace.

In general a function which belongs to space V_m also belongs to space V_{m+1}. Hence a ladder of subspaces is implied as depicted below:

$$...V_{-2} \subset V_{-1} \subset V_0 \subset V_1 \subset V_2...$$

Intuitively, we can see that as we move towards right i.e. up the ladder we are moving towards $L_2(\mathbb{R})$

$$\{\overline{\cup V_m}\}_{m\in\mathbb{Z}} = L_2(\mathbb{R})$$

The expression above, if written without the cover-line implies that it covers all the interior region whereas with closure (by cover-line) ensures the covering of boundary patches too. This talk is not of much importance though but a slight glance at it is necessary.

[11] जहदजहल्लक्षणा - वाच्यार्थस्यैकदेशं च परित्यज्यैकदेशकम्।

Now what happens when we go left towards the ladder? Movement towards leftwards implies, piecewise constant approximation over larger and larger intervals as m in $2^{-(m)}$ goes more and more negative. Now consider L_2 norm of function going towards leftwards:

$$\sum_{n=-\infty}^{\infty} |C_m(n)|^2 2^{-m}$$

where $C(\cdot)$ is approximate coefficient at resolution 2^{-m}. Now as we move towards left m becomes negative and $m \to -\infty$. Therefore L_2 norm is given by

$$2^{|m|} \sum_{n=-\infty}^{\infty} |C_m(n)|^2$$

If we require L_2 norm to converge, however for large $|m|$, $\sum_{n=-\infty}^{\infty} |C_m(n)|^2$ must be zero. That is $C_m(n) = 0 \,\forall\, n$. Hence movement towards left implies movement towards trivial subspace $\{0\}$.

$$\{\cap V_m\}_{m\in\mathbb{Z}} = \{0\}$$

One **important point** to note here is that trivial subspace is different then null subspace as null subspace does not contain any element. Thus, this NOT in compliance with 'Śūnyavāda' (शून्यवाद) of the Buddhist (बौद्ध) [12].

We say that a set of functions $\{f_1, f_2, f_3, ..., f_k, ...\}$ span a whole space if any function in that space can be represented by the linear combination of these functions.
What is function $\phi(t)$ and how does its integer translates span V_0? We may consider function $\phi(t)$ as shown in Figure 4.11.

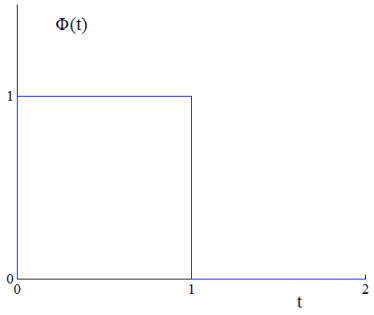


Figure 4.11: Function $\phi(t)$

Any function in V_0 can be expressed in the form

[12] As given in Vedānta Bhāṣya (2-2-31): शून्यवादिपक्षस्तु सर्व-प्रमाण-विप्रतिषिद्ध इति तन्निराकरणाय नादरः क्रियते।

$$\sum_{n\in\mathbb{Z}} C_n \phi(t-n)$$

where C_n is piecewise approximation constants and $\phi(t-n)$ are the integer translates of $\phi(t)$. Figure.4.12 shows a function belonging to V_0. It can be expressed in terms of translates of $\phi(t)$ as shown below:

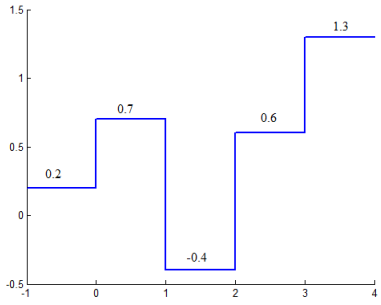


Figure 4.12: Example of a function belonging to V_0

$$0.2\phi(t+1)+0.7\phi(t)-0.4\phi(t-1)+0.6\phi(t-2)+1.3\phi(t-3)$$

Hence any space V_m can be similarly constructed using a function $\phi(2^m t)$.

$$V_m = span\{\phi(2^m t-n)\}_{n,m\in\mathbb{Z}}$$

$\phi(t)$ is called as **scaling function**(Haar MRA), also known as 'Father function'. The ladder of subspaces

$$...V_{-2}\subset V_{-1}\subset V_0\subset V_1\subset V_2...$$

with these properties is called as Multi-Resolution Analysis (MRA).

The Sarga (सर्ग) in the light of Wavelets can be looked upon as Multi-resolution Analysis (MRA). The MRA axioms and theorem are stated as follows:

4.10 Axioms of MRA

R There exists a ladder of subspaces, $...V_{-2}\subset V_{-1}\subset V_0\subset V_1\subset V_2...$ such that

1. $\{\overline{\cup V_m}\}_{m\in\mathbb{Z}} = L_2(\mathbb{R})$
2. $\{\cap V_m\}_{m\in\mathbb{Z}} = \{0\}$
3. There exists $\phi(t)$ such that, $V_0 = span\{\phi(t-n)\}_{n\in\mathbb{Z}}$
4. $\{\phi(t-n)\}_{n\in\mathbb{Z}}$ is an orthogonal set.
5. If $f(t)\in V_m$ *then* $f(2^{-m}t)\in V_0, \quad \forall m\in\mathbb{Z}$
6. If $f(t)\in V_0$ *then* $f(t-n)\in V_0, \quad \forall n\in\mathbb{Z}$

Here, $L_2(\mathbb{R})$ refer to the finest possible scale and is equivalent to the very concept of Prakṛti (प्रकृति) or Māyā (माया) attaining back the equilibrium of Sattva (सत्त्व), Raja (रज) and Tama (तम). The $\{0\}$ is the coarsest form and can be compared to Paṃca-Mahābhūta (पंच महाभूत). All the scales that translate from $L_2(\mathbb{R})$ to $\{0\}$ is the very process of evolution, that is Sarga (सर्ग).

4.11 Theorem of MRA

R Given the axioms, there exists a $\psi(\cdot) \in L_2(\mathbb{R})$, so that $\{\psi(2^m t - n)\}_{m\in\mathbb{Z}, n\in\mathbb{Z}}$ spans the $L_2(\mathbb{R})$.

The wavelet function $\psi(\cdot)$ is also called as 'Mother function' or 'Māyā function'.

The wavelet function $\psi(\cdot)$ is also called as 'Mother function'. This mother function is the Prakṛti (प्रकृति) or Māyā (माया) and is responsible for moving from one subspace to another thus creating coarser forms from finer ones or taking coarser forms towards finer ones by adding details and thus making the system comply to $L_2(\mathbb{R})$ norm.

This leads us to a sound mathematical framework which explains the crux as follows:

Analytics 4.11.1 — Spanning Spaces. In this illustration we will consider the vector space V_j spanned by the discrete scaling functions $\{\phi(2^j x - k)\}$ or Śapala Brahma (शपल ब्रह्म) function, we use this span of the vector space to project any function say $f(x)$ such that $f_j(x) \in V_j$. The projected function can be mathematically encoded as,

$$f_j(x) = \sum_k \alpha_{j,k} 2^{\frac{j}{2}} \phi(2^j x - k) \tag{4.1}$$

In this equation, $\alpha_{j,k}$ depict the approximation values as it derives the components from the low pass scaling equation as given in equation,

$$\alpha_{j,k} = \int_{-\infty}^{\infty} f(x) 2^{\frac{j}{2}} \phi(2^j x - k) dx \tag{4.2}$$

where the set $\{\phi(2^j x - k)\}$ constitutes the basis. This $f_j(x)$ represents "approximation" of the signal $f(x)$. This is exactly what is the essence of 'तदैक्षत बहुस्यां प्रजायेय।' चां.६.२.३,तै.२.६. The same is also depicted in Pañcadaśī (पञ्चदशी) [a]. In most of the de-noising applications this finds it's scope, and this set is orthogonal on $(-\infty, \infty)$ with respect to the translation, i.e.,

$$\int_{-\infty}^{\infty} 2^j \phi(2^j x - k) \phi(2^j x - k') = 0, \quad k \neq k'$$

Also,

$$\int_{-\infty}^{\infty} \phi^2(2^j x - k) dx = \frac{1}{2^j} \tag{4.3}$$

since,

$$\int_{-\infty}^{\infty} \phi^2(2^j x - k) dx = \frac{1}{2^j} \int_{-\infty}^{\infty} \phi^2(2^j x - k) \frac{1}{2^j} dx \tag{4.4}$$

$$= \frac{1}{2^j} \int_{-\infty}^{\infty} \phi^2(y - k) dy = \frac{1}{2^j} \tag{4.5}$$

where we used the change of variable $y = 2^j x - k$ and the fact that $\int_{-\infty}^{\infty} \phi^2(x)dx = 1$. So, the factor $2^{\frac{j}{2}}$ is for the normalization of $\phi(2^j x - k)$ to be $2^{\frac{j}{2}}\phi(2^j x - k)$, in order that we have,

$$\int_{-\infty}^{\infty} 2^{\frac{j}{2}}\phi(2^j x - k)2^{\frac{j}{2}}\phi(2^j x - k')dx = \begin{cases} 0, & k \neq k' \\ 1, & k = k \end{cases} \tag{4.6}$$

We mention that the orthogonality of the scaling functions on $(-\infty, \infty)$ with respect to translations, is a very basic requirement in the (usual) definition of the multi-resolution analysis [b].

We know that $V_j \in V_{j+1}$ as depicted in Figure (4.13), and that V_{j+1} has better refinement than V_j. This "difference" (as only a refinement and not an approximation like $f_j(x)$ in (4.1) of the signal!) is a subset of V_{j+1} spanned by the discrete wavelets of the subspace W_j, $g_j(x) \in W_j$ or the Māyā (माया) space,

$$g_j(x) = \sum_k \beta_{j,k} 2^{\frac{j}{2}} \psi(2^j x - k) \tag{4.7}$$

This spanning of the spaces of Māyā (माया) projections also happens in the dyadic style. Māyā (माया) projections are depictions of the details captured and in terms of signal processing it is termed as High-Pass-Filtering! Because the Māyā (माया) projections are capable of spanning various resolutions, from coarsest to the finest, Ṛgveda (ऋग्वेद) in (1-32-4) proclaims - मायिनाममिनाः प्रोत मायाः। It should be noted that the word 'Māyā (माया)' appears several times in Ṛgveda (ऋग्वेद). Some scholars have taken the objection that Ādi Śaṅkaraacaarya (आदि शङ्कराचार्य) invented Māyā (माया) to propose his doctrine of Māyā (माया) which is popularly called as Māyāvāda (मायावाद) or Vivartavāda (विवर्तवाद). Obviously this is not true and the doctrine put forth by Ādi Śaṅkaraacaarya (आदि शङ्कराचार्य) is honestly consistent with the Vedas. Ādi Śaṅkaraacaarya (आदि शङ्कराचार्य) has certainly elaborated on the concept of Māyā (माया) in BrahmasūtraBhāṣya (ब्रह्मसूत्रभाष्य) - यथा मायाविनः चर्मखड्गधरात् सूत्रेण आकाशं अधिरोहतः स एव मायावी परमार्थरूपो भूमिष्ठोऽन्यः। in (1-1-17) and एक एव परमेश्वरः कूटस्थनित्यो विज्ञानधातुः अविद्यया मायया मयाविवत् अनेकधा विभाव्यते। in (1-3-19). This exactly what the dot product of $\beta_{j,k} \cdot 2^{\frac{j}{2}} \psi(2^j x - k)$ indicates.

$$\beta_{j,k} = \int_{-\infty}^{\infty} f(x) 2^{\frac{j}{2}} \psi(2^j x - k) \tag{4.8}$$

The basis $\{\psi(2^j x - k)\}$ of this vector space W_j are always orthogonal to the scaling functions $\{\phi(2^j x - k)\}$ of V_j on $(-\infty, \infty)$,

$$\int_{-\infty}^{\infty} 2^{\frac{j}{2}}\phi(2^j x - k)2^{\frac{j}{2}}\psi(2^j x - k')dx = \delta_{k,k'} \tag{4.9}$$

This relation resembles the orthogonality of the cosine functions to the sine functions of the Fourier series.

An example again is that of Haar function and their associated wavelets, where they are orthogonal even when $k = k'$, as can be seen from figure (4.14), where their product is ,for example $\phi(x-1)\psi(x-2)$ or $\phi(x-1)\psi(x-1)$, is zero on $(-\infty, \infty)$. With this orthogonality of the basis of V_j to those of W_j, we have W_j as the orthogonal complement of V_j in V_{j+1}, $V_{j+1} = V_j \oplus W_j$. This can be done in the same way for $V_j = V_{j-1} \oplus W_{j-1}$, whereas $V_{j+1} = V_{j-1} \oplus W_{j-1} \oplus W_j$. Then this

process can be continued until we reach a coarse (or blurred) resolution, for example, such that of V_0 with scale $\frac{1}{2^0} = 1$,

$$V_{j+1} = V_0 \oplus W_0 \oplus W_1 \oplus W_2 \oplus ... \oplus W_{j-1} \oplus W_j \tag{4.10}$$

This is a mathematical form of जीवो ब्रह्मैव ना परः । The orthogonal addition in lower subspace between V_j and W_j leads to V_{j+1} formation!

Figure (4.14) shows schematically this relation with the blurred approximation in V_0 of scale $l_0 = 1$, and the refinements that are added to it by W_0, W_1, W_2 and W_3. We must note that, for example, V_1 and W_1 are two subspaces in V_2, where the W_2 basis span the difference between V_2 and V_1.

In the next illustration we show the decomposition $f_0(x) \in V_0$ of the function $f(x) = x,\ 0 < x < 3$ as a rough approximation of $f(x)$, then wrote its decomposition (refinement) as $g_0(x) \in W_0$. We showed that the latter decomposition $g_0(x)$ is not an approximation to the function $f(x) = x,\ 0 < x < 3$, but it represented a helping hand to $f_0(x) \in V_0$ at the scale $l_0 = 1$ where it added to it a bit of a refinement to advance this approximation $f_0(x)$ to the more refined one $f_1(x) \in V_1$ at the smaller scale $l_1 = \frac{1}{2}$. This is the essence of W_0 being the orthogonal complement of V_0 in V_1. Both V_0 and W_0 are subspaces of V_1 and $V_1 = V_0 \oplus W_0$, where the basis of W_1 span the difference between V_1 and V_0. Another way of putting this is that, for having the refinement at the lower scale $l_1 = \frac{1}{2}$, that help us go from V_0 at scale $l_0 = 1$ to V_1 at the scale $\frac{1}{2}$, $V_0 \in V_1$, we can go to the basis of W_0 at the same scale $l_0 = 1$ to span the difference between the spaces V_1 and V_2. This alternative means that while we work with the scale $l_0 = 1$ for V_0 and W_0, we are getting the equivalent refinement of being with the smaller scale $l_1 = \frac{1}{2}$ in V_1. Well, the credit must go to the little details (refinement) in the wavelets series (4.8) versus its associated scaling function series of (4.2) for $j = 0$. This can be seen clearly, where the Haar basic wavelet,

$$\psi(t) = \begin{cases} 1, & 0 \le x < \frac{1}{2} \\ -1, & \frac{1}{2} \le x < 1 \\ 0, & \text{otherwise} \end{cases} \tag{4.11}$$

has more structure (refinement-details) than that of its associated scaling function,

$$\phi(t) = \begin{cases} 1, & 0 \le x < 1 \\ 0, & \text{otherwise} \end{cases} \tag{4.12}$$

[a]चिदानन्दमयब्रह्मप्रतिबिम्बसमन्विता । पञ्चदशी.1.15

[b]तमः प्रकाशवत् - विरुद्ध स्वभावयोः in the preface or उपोद्घात of शारीरक भाष्य by आदि शङ्कराचार्य

Analytics 4.11.2 — A brief look at implementing signal decomposition with filters. In line of this observation that the wavelets (Māyā or माया) decomposition brings refinement to that of the scaling functions (blurred) (Śapala Brahma or शपल ब्रह्म) approximation of the signal in $f_0(x) + g_0(x) = f_1(x) \in V_1$, we may venture to make this comparison in terms of the "frequency content" of the signal as it is measured by the inner product of $< f(x), \phi(x) >$ and $< f(x), \psi(x) >$ of the function $f(x)$ and the Haar scaling function and its associated wavelet (Māyā or माया function) respectively. The simplistic observation is that the Haar function $\phi(x),\ 0 < x < 1$ resembles the behavior of $\cos 0x = 1$ on $(0, \pi)$, while $\psi(x), 0 \le x < 1$ with its oscillation-like on

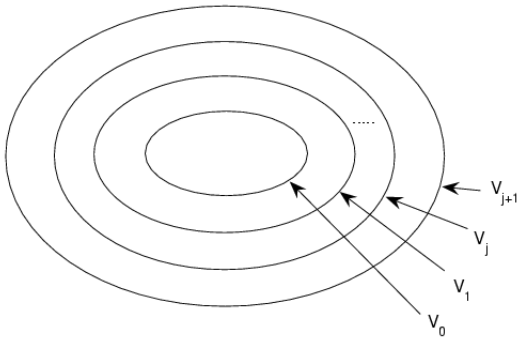


Figure 4.13: Nested subspaces spanned by the (discrete) scaling function $\phi(x), \phi(2x-k), ..., \phi(2^j x - k), \phi(2^{j+1}x-k)$

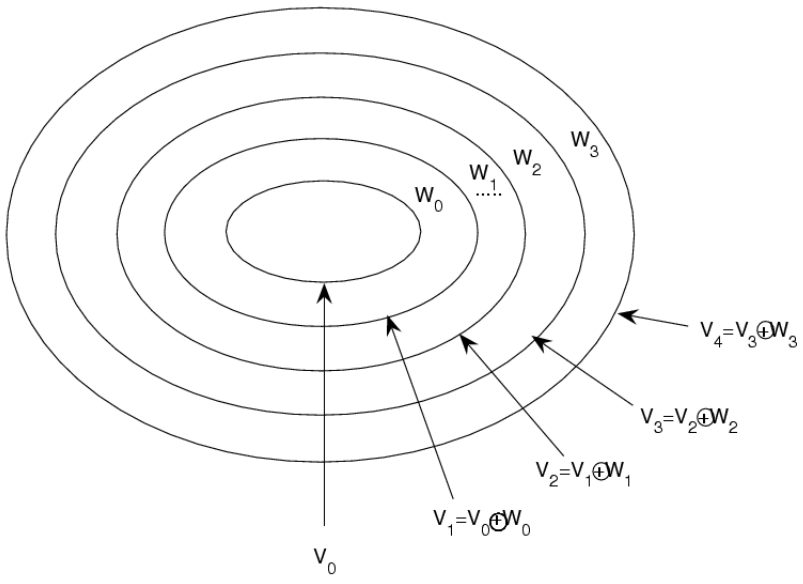


Figure 4.14: Wavelets subspaces W_j as the orthogonal components of V_j in V_{j+1}, $V_{j+1} = V_j \oplus W_j$

Description	Type of ख्याति	Philosophical School
स्मृतिरूपः परत्र पूर्वदृष्टावभासः।	अनिर्वचनीय ख्याति	अद्वैत वेदान्ती
अन्यत्रान्यधर्माध्यासः।	अन्यथाख्याति	नैयायिक
अन्यत्रान्यधर्माध्यासः।	आत्मख्याति	योगाचार बौद्ध
यत्र यदध्यासः तद्विवेकाग्रहनिबन्धनो भ्रमः।	अख्याति	मीमांसक
यत्र यदध्यासः तस्यैव विपरीतधर्मत्वकल्पना।	असत् ख्याति	माध्यमिक बौद्ध

Table 4.2: Types of Khyāti

$[0,1)$ resembles $\cos x$ on $(0,\pi)$, which is with higher frequency $n=1$ than that of 0 for $\cos 0x$. Now, the coefficients $\alpha_{0,0}$ and $\beta_{0,0}$, as the inner products $< f(x),\phi(x) >$ (Śapala Brahma or शपल ब्रह्म projections) and $< f(x),\psi(x) >$ (Māyā or माया projections), respectively, measure the frequency content in the signal- quoting Fourier series analysis. Hence $< f(x),\psi(x) >$ is expected to indicate a frequency content of frequency 1, while $< f(x),\phi(x) >$ would indicate a frequency content of zero frequency. Hence, the refinement we get from $< f(x),\psi(x) >$ versus that of $< f(x),\psi(x) >$.

Another more accurate way of interpreting $< f(x),\phi(x) >$ and $< f(x),\psi(x) >$ is that for a continuous function $f(x)$ on $[0,1)$,

$$< f(x),\phi(x) >= \int_{-\infty}^{\infty} f(x)\ 1\ dx = \int_0^1 f(x)dx, \tag{4.13}$$

which is the "average" of $f(x)$. So, $< f(x),\phi(x) >$ gives a very coarse "blurred" approximation of $f(x)$ as just "the average" of $f(x)$ on $[0,1)$. These are Śapala Brahma or शपल ब्रह्म projections.

On the other hand,

$$\begin{aligned} < f(x),\psi(x) > &= \int_{-\infty}^{\infty} f(x)\psi(x)dx \\ &= \int_0^{\frac{1}{2}} f(x)\ 1\ dx + \int_{\frac{1}{2}}^1 f(x)\ (-1)\ dx \end{aligned}$$

$$< f(x),\psi(x) >= \int_0^{\frac{1}{2}} f(x)\ 1\ dx - \frac{1}{2}\int_{\frac{1}{2}}^1 f(x)\ dx \tag{4.14}$$

which is the difference between the two averages of $f(x)$ on $[0,\frac{1}{2})$ and $[\frac{1}{2},1)$. Hence, while $< f(x),\phi(x) >$ is satisfied with just the average of $f(x)$ on $[0,1)$, $< f(x),\psi(x) >$ goes much farther than that searching for "differences", "variations", "refinements" or "fine structure" in the signal. These are Māyā or माया projections. So, again, the mere blurred averaging of $< f(x),\phi(x) >$ or Śapala Brahma or शपल ब्रह्म projections versus the more detailed differential of $< f(x),\psi(x) >$ in the signal or Māyā or माया projections, may still be associated with low and high frequency by contents, respectively.

Long before the advent of the wavelets analysis about a quarter of a century ago, now and forever, electrical engineers work with the frequency content of the signal. Before wavelets, their measure of frequency contents was in the Fourier series coefficients, $a_n = \frac{1}{\pi} < f(x), \cos nx >$, $b_n = \frac{1}{\pi} < f(x), \sin nx >$, $-\pi < x < \pi$ as, again, inner products. Also, for $f(x) \in (-\infty, \infty)$, the Fourier integral (transform), $F(w) = \frac{1}{2\pi} < f(x), e^{-i\omega x} > = < f(x), e^{-i2\pi\nu x} >$, where ω is the frequency in radians per seconds and ν is in cycles per seconds (hertz). The periodic signals are subjected to Fourier series analysis and aperiodic signals will be subjected to Fourier Transforms Techniques.

At this time we may say that the electronics engineering method of looking for the high frequencies in a signal is to use what is called a "high pass" filter or now Māyā or माया projections. This means that such filters stop all frequencies below certain frequency ω_0, and allow only the frequencies above ω_0 with a band (ω_0, ω_n), where ω_n is the highest frequency anticipated in the signal. For the low frequency content below ω_0, they have a "low pass" filter that allows only such low frequencies or Śapala Brahma or शपल ब्रह्म projections. Now in the context of $< f(x), \psi(x) >$ searching for the "differences" or higher frequencies than that of $< f(x), \phi(x) >$ the content with the average, blurred or low frequency. Electrical engineers jumped at implementing such operations with "high pass" or Māyā or माया and "low pass" or Śapala Brahma or शपल ब्रह्म filters respectively. Let us assume that we have a signal that may have a refinement close to the scale $l_1 = \frac{1}{2}$. So, we approximate it in the space V_1 with its scaling functions $\{\phi_{1,0}, \phi_{1,1}\}$ and their respective coefficients $\alpha_{1,0}, \alpha_{1,1}$. This is in line with approaching Sarga (सर्ग). Since these functions use is for approximating the signal by $f_1(x)$ in V_1, we know the basis used $\phi_{1,0}, \phi_{1,1} = \{\sqrt{2}\phi(2x), \sqrt{2}\phi(2x-1)\}$. So, we will indicate the $f_1(x)$ approximation by the coefficients used for $\{\alpha_{1,0}, \alpha_{1,1}\}$ in the approximation (ब्रह्म) or decomposition (माया) series. We do the same for $g_1(x) \in W_1$, where we indicate it by its coefficients $\{\beta_{1,0}, \beta_{1,1}\}$ of the wavelets $\{\sqrt{2}\psi(2x), \sqrt{2}\psi(2x-1)\}$. We take it that the coefficients $\{\alpha_{1,0}, \alpha_{1,1}\}$ of the scaling function represent an average or "blurred" decomposition of the signal, while the wavelets coefficients $\{\beta_{1,0}, \beta_{1,1}\}$ represent the search for "differences" or changes in the signal (as with माया or प्रकृति). To have these two different characters that gives the indication of the low frequency associated with $\{\alpha_{1,0}, \alpha_{1,1}\}$ and a high frequency associated with the differences of $\{\beta_{1,0}, \beta_{1,1}\}$, the signal is passed through two parallel low pass and high pass filters. Thus, Śapala Brahma (शपल ब्रह्म) and Māyā (माया) work in tandem.

The output of the low pass filter, as expected, is associated with the averages of $\{\alpha_{1,0}, \alpha_{1,1}\}$ of the scaling शपल ब्रह्म functions, while the output of the high pass filter is associated with the (differential) coefficients $\{\beta_{1,0}, \beta_{1,1}\}$ of the wavelets माया function.

This operation is represented schematically as shown in Figure (4.15). We note that if the signal has, for example, 256 samples, then the outputs of the two (parallel) filters add up to double of that,i.e., 512 samples. What is done in this case is to cancel half of the samples of each output to end up with the same number (256) of the input (to the two filters) samples. The usual way of doing this is to throw away every other sample from the output of each filter to end up with 128 samples output for each sample, a total of 256, the same number of input samples. This process of cutting the output samples to a half, is called "down sampling", and is designated by the sign $\downarrow$. We shall refer to the operations of the low and high pass filters by L and H, respectively.

As seen in Figure (4.15), while we have only averages associated with the coefficients $\{\alpha_{1,0}, \alpha_{1,1}\}$ of the scaling functions in V_1, the output of the low pass filter still gave a "blurred" average associated with $\alpha_{0,0}$ of $\phi_{0,0}$, while the output of the high pass filter produced differential associated with $\beta_{0,0}$ of the wavelet $\psi_{0,0}$. This is the essence, in a very practical way, of saying that

$V_1 = V_0 \oplus W_0$.

This also very strongly establishes Adhyāsavāda (अध्यासवाद) or Vivartavāda (विवर्तवाद) of as per Vedānta school of thought. The various thoughts on this can be reflected as given in Table (4.2).

Here अनिर्वचनीय ख्याति gets subtly established. As per Advaita Vedānta, the process of gaining knowledge (प्रमा = यथार्थ ज्ञान) happens because of Caitanya (चैतन्य). This Caitanya (चैतन्य) is of four types and gets captured neatly in this framework as follows:

1. प्रमातृचैतन्य - $< \alpha_{j,k}, f_j >$
2. प्रमाणचैतन्य - $< \alpha_{j,k}, g_j >$
3. विषयचैतन्य - $< \beta_{j,k}, f_j >$
4. फलचैतन्य - $< \beta_{j,k}, g_j >$

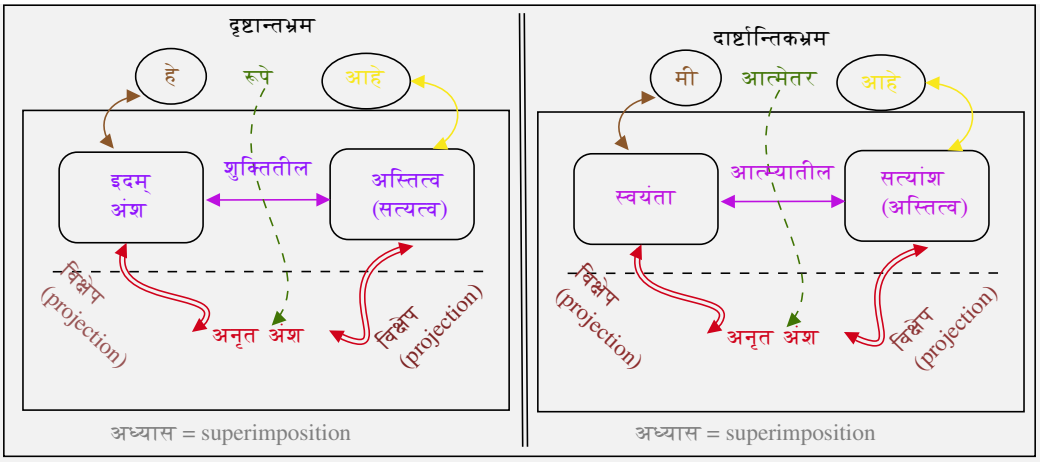
All other types of ख्याति are naturally refuted thus. What we have in Figure (4.15) is the decomposition process of $f_1(x) \in V_1$ via its representation of the sum of the two outputs of the low and high pass filters. Such a decomposition may be stored or transmitted. In the latter case, we receive the two outputs of Figure (4.15), and we are to reconstruct the original signal. For this, the reverse process of Figure (4.15) is used as shown in Figure (4.16). However, when we have such a two outputs with, for example, 128 samples each, they will be inputs together as only 128 and not the original 256 samples of the input of the decomposition. For this an "up sampling" $\uparrow$ by a factor of 2 is used to recover the number 256. This is done by inserting 128 zeros in between the 128 samples. The two operations in Figure (4.16) of decomposition, then the reconstruction in Figure (4.17) constitute a simple example of what is called "quadrature mirror filter pairs".

In Figure we showed the decomposition as well as the reconstruction of the signal $f_2(x) \in V_2$ via the (quadrature mirror filter) pairs.

As we had commented on Figure (4.15), Figure (4.17) depicts the first stage of the two filters is expressed as $V_2 = V_1 \oplus W_1$, while the second stage of the decomposition via the second two filters is expressed as $V_1 = V_0 \oplus W_0$. Hence the output of the two sets of the parallel low pass and high pass filters becomes $V_2 = V_0 \oplus W_0 \oplus W_1$. This means that the details in V_2 are formed via the projections of $f(x)$ onto the wavelets spaces W_0 and W_1 with scales of 1 and $\frac{1}{2}$, respectively. The scaling functions role, on the other hand, is only a coarse (blurred) projection of the signal onto their space V_0 with its large scale of 1 in the above case.

So, for the general case when a signal is suspected to need a very fine projection, for example $l_5 = \frac{1}{3^2}$, then we start with $f_5(x) \in V_5$, and the above decomposition goes as $V_5 = V_0 \oplus W_0 \oplus W_1 \oplus W_2 \oplus W_3 \oplus W_4$, which requires five low and high pass filter pairs.

More details on defining filters including the low and high pass ones, are can be understood with the following depiction of Adhyāsa (अध्यास).



This concept of Adhyāsa is very powerful and once again it gets established through अनिर्वचनीय ख्याति as: सत्त्वेनासत्त्वेन चानिर्वचनीयस्य रजतादेः ख्यातिः प्रतीतिः अनिर्वचनीयख्यातिः ।. Because of this, the ladder structure of the nested sub-sets holds and we get the complete MRA (Multi Resolution Analysis) framework.

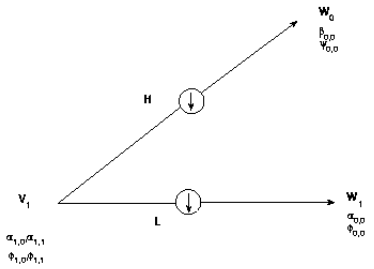


Figure 4.15: The outputs $f_0(t) \in V_0$ and $g_0(t) \in W_0$ of the low and high pass filters respectively (after down sampling)

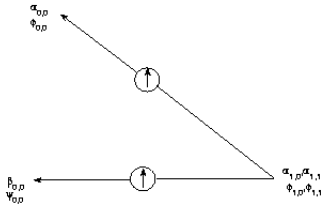


Figure 4.16: The reconstruction part of the (quadrature mirror) filter pairs to implement the scaling function wavelets reconstruction of the signal $f_1(x) \in V_1$.

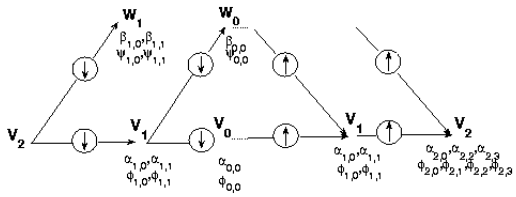


Figure 4.17: Decomposition and reconstruction of $f_2(x) \in V_2$ - A special case of a "quadrature mirror filter pairs".

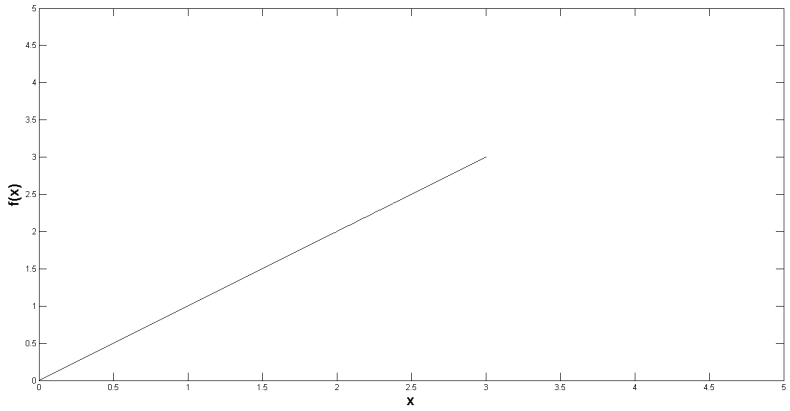


Figure 4.18: Signal f(x)

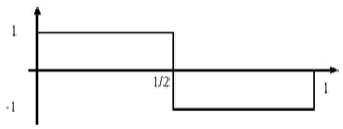


Figure 4.19: Haar Wavelet Function

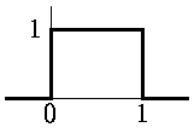


Figure 4.20: Haar scaling Function

The conceptual part of Sarga (सर्ग) is well captured now. To understand the practical part and to enable us to understand the actual process, we uncover one complete signal decomposition cycle by invoking the zoom-in and zoom-out features of Wavelet function or the Prakṛti (प्रकृति).

Analytics 4.11.3 — Zoom-in and Zoom-out. We have seen that wavelet transform decomposes signal into two separate series and this is in line with the basic understanding of the notion of 'Brahma' (ब्रह्म) and 'Māyā' (माया). One is a single series which represents coarse version which leads to scaling function and corresponds with 'Brahma' (ब्रह्म), popularly known as the father function. Other is a double series which represents a refined version which leads to wavelet function and corresponds to 'Māyā' (माया), popularly known as mother wavelet. We also build on top of the idea of nested subspaces, an MRA and two band filter bank to realize an MRA. The major problem regarding an MRA was that while analyzing a signal we move down the ladder in an MRA. However in many applications moving up the ladder is desired. Thus an alternative framework is needed. So let us start defining that framework.
Whole concept of MRA is in a way based on the idea of nested subspaces. This gives the entire wavelet structure ability to 'zoom-in' and 'zoom-out' of any signal under analysis. We can bring about this essence using two mathematical equations.

1. Equation of scaling function,

$$\phi(t) = \sqrt{2}\sum_{k} h_k \phi(2t-k) \tag{4.15}$$

In above equation we can clearly see that $\phi(t)$ belongs to subspace V_0. Similarly we can see that $\phi(2t-k)$ belongs to subspace V_1. Thus idea of nested subspaces is captured in the above equation.

2. Equation of Wavelet function,

$$\psi(t) = \sqrt{2}\sum_{k} g_k \phi(2t-k) \tag{4.16}$$

Similar to the case seen above we can see that $\psi(t)$ belongs to subspace W_0 and $\phi(2t-k)$ belongs to V_1.

From above two equations we can see that,

$$V_j = V_j - 1 \oplus W_j - 1$$

Above equation leads us again to the idea of subspaces. If we start at V_0 and go on adding details then at some stage we would be able to go tantalizingly close to the signal. Let's now see how to achieve that,

Let a function be,

$f_j(x) \in V_j$, scale=$\frac{1}{2^j}$.
To span these space V_j the basis function would be $2^{j/2}\phi(2^j x - k)_k$

Here k is the translational parameter and $2^{j/2}$ is the normalizing factor to convert orthogonal basis into an orthonormal basis.
Thus we can write,

$$f_j(x) = \sum_k (\alpha_{j,k} 2^{j/2} \phi(2^j x - k))$$

Alpha can be calculated by as,

$$\alpha_{j,k} = \int_{-\infty}^{+\infty} f_j(x) 2^{j/2} \phi(2^j x - k) dx$$

Similarly for W subspaces,

$g_j(x) \in W_j$, scale = $\frac{1}{2^j}$

The basis would now be,

$2^{j/2}\psi(2^j x - k)_k$

Also we can write,

$$g_j(x) = \sum_k (\beta_{j,k} 2^{j/2} \psi(2^j x - k))$$
$$\beta_{j,k} = \int_{-\infty}^{+\infty} g_j(x) 2^{j/2} \psi(2^j x - k) dx$$

β values here would give us the details which are required to move from one subspace to another.

Now to understand how the framework we designed is useful in moving up the ladder let us consider a problem.

Consider a signal,

$$f(x) = \begin{cases} x & 0 \leq x \leq 3, \\ 0 & \text{elsewhere} \end{cases}$$

The function $f(x)$ is shown in figure (4.18).
Objectives:

1. To find $f_0(x) \in V_0$

2. To find $g_0(x) \in W_0$

3. To find $f_1(x) \in V_1$ by $f_1(x) = f_0(x) \oplus g_0(x)$. This would be moving up the ladder since we are getting $f_1(x)$ from $f_0(x)$ and $g_0(x)$.

4. Ultimately to prove $V_0 \oplus W_0 = V_1$

Firstly let us find $f_0(x) \in V_0$, j=0 Scale= $\frac{1}{2^j}$=$\frac{1}{2^0}$=1. In, $(2^{j/2}\phi(2^j x - k))_k$,

if we put j=0 the basis function would be, $\psi(x-k)_k$. The projection would be,
$f_0(x) = \sum_{k=0}^{2}(\alpha_{0,k}\phi(x-k))$

For k=0,

$$\alpha_{0,0} = \int_{-\infty}^{+\infty} f_0(x)\phi(x)dx$$

The interesting thing to note is, here we can choose our own scaling function ϕ which is not the case with Fourier or any other conventional transforms discussed in the last lecture. In the conventional transforms the basis function is fixed and is exponential.
Here we would be using Haar scaling and wavelet functions shown in figure (4.19) and figure (4.20).

For k=0 the equation would result in,

$$\alpha_{0,0} = \int_0^1 x dx$$
$$\alpha_{0,0} = \frac{x^2}{2}\Big|_0^1$$
$$\alpha_{0,0} = \frac{1}{2}$$

Correspondingly,

$$\alpha_{0,1} = \int_1^2 x\phi(x-1)dx$$
$$\alpha_{0,1} = \int_1^2 xdx$$
$$\alpha_{0,1} = \frac{3}{2}$$

Similarly,

$$\alpha_{0,2} = \int_2^3 x\phi(x-2)dx$$
$$\alpha_{0,2} = \int_2^3 xdx$$
$$\alpha_{0,2} = \frac{5}{2}$$

Now we can write,

$$f_0(x) = \frac{1}{2}\phi(x) + \frac{3}{2}\phi(x-1) + \frac{5}{2}\phi(x-2) \tag{4.17}$$

This is how we can find out projections of signal f in space V_0. The coefficients of $f_0(x)$ represent the approximations of the signal. This can be seen in figure (4.21).

Now the next task is to find the projections $g_0(x)$ of the signal on W_0. Let, $g_0(x) \in W_0$, scale =1. The basis function would be, $2^{j/2}\psi(2^j x-k)_k$.
We will again make use of Haar wavelet function. Thus as j=0, the basis function would now reduce to, $\psi(x-k)_k$.
Thus we can write,

$$g_0(x) = \sum_k (\beta_{0,k}\psi(x-k))$$
$$\beta_{0,k} = \int_{-\infty}^{+\infty} g_0(x)\psi(x-k)dx$$

As the projections are taken in W_0 subspace these β values will now represent all the details. Similar to the case of α values, from above equation we can find out corresponding β values as follows,

$$\beta_{0,0} = \int_{-\infty}^{+\infty} g_0(x)\psi(x)dx$$

$$\beta_{0,0} = \int_0^1 x\psi(x)dx$$

$$\beta_{0,0} = \int_0^{1/2} (1)xdx + \int_{1/2}^1 x(-1)dx$$

$$\beta_{0,0} = \frac{1}{8} - \frac{1}{2} + \frac{1}{8}$$

$$\beta_{0,0} = \frac{-1}{4}$$

$$\beta_{0,1} = \int_1^2 x\psi(x-1)dx$$

$$\beta_{0,1} = \int_1^{3/2} (1)xdx + \int_{3/2}^2 x(-1)dx$$

$$\beta_{0,1} = \frac{-1}{4}$$

Similarly,

$$\beta_(0,2) = \frac{-1}{4}$$

Thus we can write,

$$g_0(x) = \beta_{0,0}\psi(x) + \beta_{0,1}\psi(x-1) + \beta_{0,2}\psi(x-2)$$

$$g_0(x) = \frac{-1}{4}\psi(x) + \frac{-1}{4}\psi(x-1) + \frac{-1}{4}\psi(x-2) \tag{4.18}$$

Hence we can see that all the β have the same value $\frac{-1}{4}$. We will discuss that later on in the chapter. The projections $g_0(x)$ are plotted along $f_0(x)$ in figure (4.22).

Now lets find out projections on V_1 subspace. Let, $f_1(x)\varepsilon V_1$, j=1, scale= $\frac{1}{2^j} = \frac{1}{2}$.
The basis function would now change to, $2^{j/2}\phi(2^j x-k)_k$. As j=1, $\sqrt{2}\phi(2x-k)_k$

Correspondingly we can write,

$$f_1(x) = \sum_{k=0}^{5} (\alpha_{1,k}\sqrt{2}\phi(2x-k))$$

At the point the α values obtained from above equation should match numerically with the values obtained by orthogonally adding α values in V_0 with β values in W_0. If these values match we can say that the framework we developed is definitely working and we would be able to move up the ladder.

So let's now calculate $\alpha_{1,k}$ values.

$$\alpha_{1,0} = \int_{-\infty}^{+\infty} f_1(x)\sqrt{2}\phi(2x)dx$$
$$\alpha_{1,0} = \int_{0}^{1/2} f_1(x)\sqrt{2}\phi(2x)dx$$
$$\alpha_{1,0} = \int_{0}^{1/2} f_1(x)\sqrt{2}\phi(2x)dx$$

From figure (4.23),

$$\alpha_{1,0} = \sqrt{2}\int_{0}^{1/2} x(1)dx$$
$$\alpha_{1,0} = \sqrt{2}\frac{1}{8}$$
$$\alpha_{1,0} = \frac{1}{4\sqrt{2}}$$

From figure (4.22) we can calculate $\alpha_{1,0}$ as,

$$\alpha_{1,0} = \alpha_{0,0}^{-} + \beta_{0,0}^{-}$$
$$\alpha_{1,0} = \frac{1}{2} - \frac{1}{4}$$
$$\alpha_{1,0} = \frac{1}{4}$$

The above value matches the previously calculated one with an exception of $\sqrt{2}$ which would eventually be getting cancelled since we have normalized the basis basis functions. We can see this with the help of following,

$$f_1(x) = \sum_{k=0}^{5} (\alpha_{1,k}\sqrt{2}\phi(2x-k))$$

For k=0,

$$f_1(x) = \alpha_{1,0}\sqrt{2}\phi(2x)$$
$$f_1(x) = \frac{1}{4\sqrt{2}}\sqrt{2}\phi(2x)$$
$$f_1(x) = \frac{1}{4}\phi(x)$$

Similar to $\alpha_{1,0}$ we can calculate other α values.

$$\alpha_{1,1} = \frac{3}{4}$$
$$\alpha_{1,2} = \frac{5}{4}$$
$$\alpha_{1,3} = \frac{7}{4}$$
$$\alpha_{1,4} = \frac{9}{4}$$
$$\alpha_{1,5} = \frac{11}{4}$$

Thus we can conclude that approximations obtained in V_1 are better than those obtained in V_0.

Hence this is a mechanism through which we can not only think of moving up the ladder but also think of making the choice of the scale and translation parameter and then specifically zoom on to a particular point in the signal or function which is of greater importance. This framework enables the analyzer to either zoom-in onto a specific part of the signal or zoom-out to understand the big picture. This big picture can be glanced in figure (4.24).

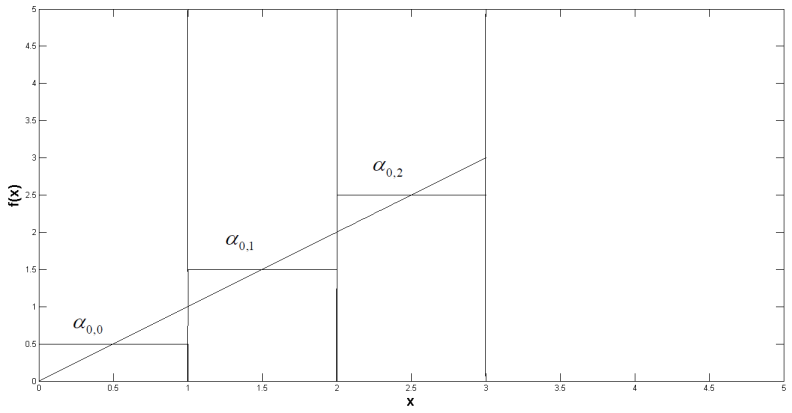


Figure 4.21: Plotting of $f_0(x)$ coefficients

The following MATLAB code presents the analysis carried out in the previous illustration.

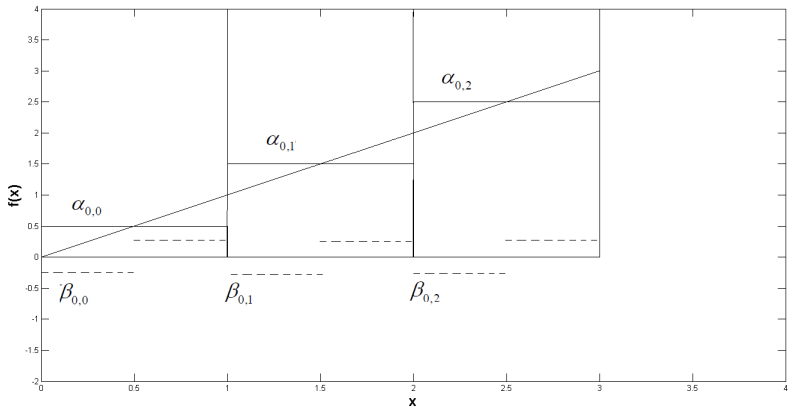


Figure 4.22: Plotting of $g_0(x)$ and $f_0(x)$ coefficients

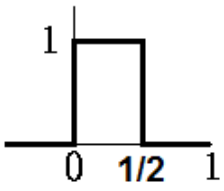


Figure 4.23: $\phi(2x)$

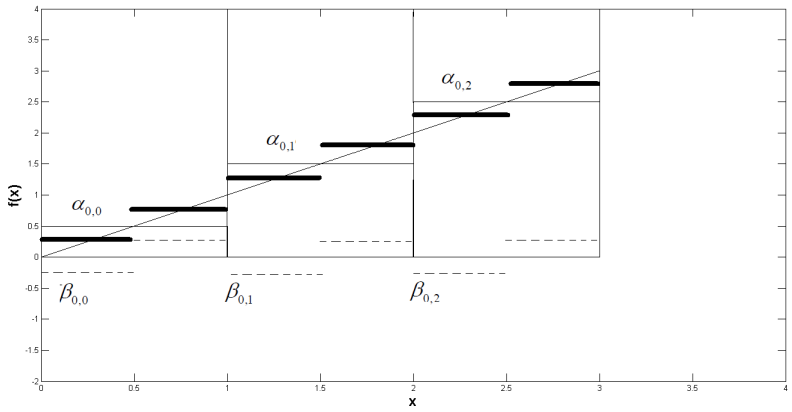


Figure 4.24: Plotting of α values of $f_1(x)$, Note - The thick black lines are α values of $f_1(x)$

Analytics 4.11.4 — MATLAB code: Fourier for Non-Stationary Signals. \\

```
%***********************************************
%***********************************************
% MATLAB code to understand MRA (MultiResolution Analysis)
% example: To be accompanied with thesis by
% by Aditya Abhyankar
%***********************************************
%***********************************************
clear all;close all;clc;

x0=0; % Lower limit of signal
x1=3; % Upper limit of signal

level=9; % Resolution of the signal
x=x0:1/100:x1-1/2^level;

alpha00=1/2;alpha01=3/2; alpha02=5/2;% alpha_0,k values for V0
beta0=1/4; beta1=-1/4; % beta_0,k values for W0

f=x; % The given function
plot(f,x,'k','LineWidth',2); axis([0 4 -1 4]); grid on; hold on;
% Ploting of projections in V0
plot(x(1:100),alpha00, '+b','LineWidth',8);
plot(x(101:200),alpha01, '+b', 'LineWidth',8);
plot(x(201:300),alpha02, '+b','LineWidth',8); hold on;
% Ploting of projections in W0
plot(x(1:50),beta1, '*g','LineWidth',8); plot(x(51:100),beta0,
'*g','LineWidth',8); plot(x(101:150),beta1, '*g','LineWidth',8);
plot(x(151:200),beta0, '*g','LineWidth',8); plot(x(201:250),beta1,
'*g','LineWidth',8); plot(x(251:300),beta0, '*g','LineWidth',8);
hold on;
alpha10=1/4;alpha11=3/4; alpha12=5/4; % alpha_1,k values for V1
alpha13=7/4;alpha14=9/4; alpha15=11/4;
% Ploting of projections in V1
plot(x(1:50),alpha10, 'or','LineWidth',1); plot(x(51:100),alpha11,
'or','LineWidth',1); plot(x(101:150),alpha12, 'or','LineWidth',1);
plot(x(151:200),alpha13, 'or','LineWidth',1);
plot(x(201:250),alpha14, 'or','LineWidth',1);
plot(x(251:300),alpha15, 'or','LineWidth',1);
title('Multi-Resolution Example'); save test_f.mat f;
% End
```

The saved function can be opened in the wavelet toolbox of MATLAB by typing 'wavemenu' on the command prompt and loading the test signal and doing analysis using wavelet of the reader's choice. The output is shown in the figure(4.25).

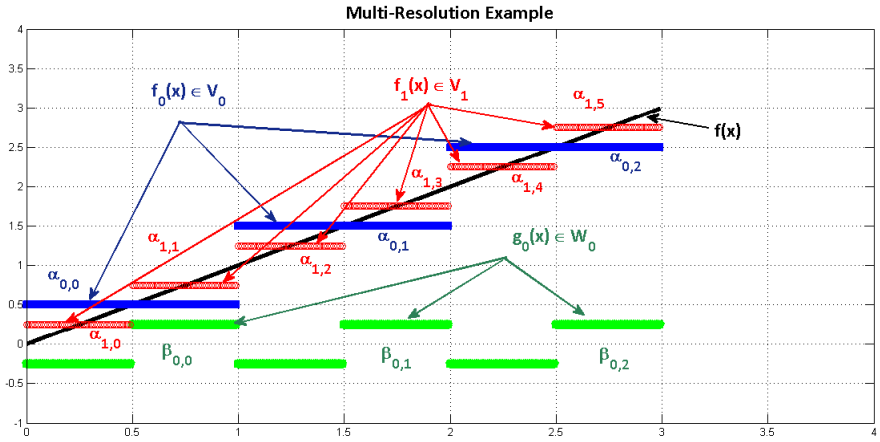
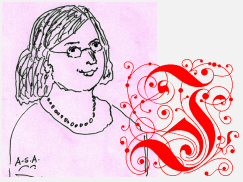


Figure 4.25: MRA exercise. Black line indicates the function $f(x)$, blue projections indicate $f_0(x) \in V_0$, green projections indicate $g_0(x) \in W_0$, red projections indicate $f_1(x) \in V_1$

Along with Sarga (सर्ग), PratiSarga (प्रतिसर्ग) is extremely important. Both the procedures are possible in the form of either moving up or down the ladder of nested sub-sets. The same is illustrated further in this chapter.

4.12 Daubechies Filters

The salient feature of Daubechies filter bank is that its construction of depends on addition of polynomial of higher and higher degree in filter transfer function. To be specific, more and more $(1-z^{-1})$ terms are utilized in high pass analysis filter bank.



ngrid Daubechies

(born 17 August 1954) is a Belgian physicist and mathematician. Between 2004 and 2011 she was the William R. Kenan, Jr. Professor in the mathematics and applied mathematics departments at Princeton University. In January 2011 she moved to Duke University as a Professor in mathematics. She is the first woman president of the International Mathematical Union (20112014). She is best known for her work with wavelets in image compression. At Courant in 1986 she made her best-known discovery: based on quadrature mirror filter-technology she constructed compactly supported continuous wavelets that would require only a finite amount of processing, in this way enabling wavelet theory to enter the realm of digital signal processing.

Impulse response of Daubechies analysis low pass filter

The first member of the Daubechies family is the Haar filter bank itself. In the second member of Daubechies family the analysis side high pass filter has factor of $(1-z^{-1})^2$. Now high pass filter of analysis side is of form $z^{-D}H_0(-z^{-1})$, where $H_0(z)$ is the analysis side low pass filter. So $H_0(z)$ should have a factor of $(1+z^{-1})^2$.

Analytics 4.12.1 — D2 calculations. It can be recalled that in the Daubechies family the filter lengths are always even. So for the second member of the Daubechies family, filter length will be 4 and the order will be 3. So $H_0(z)$ has 3 zeros. Two of them are already specified to be at $z=-1$. The third zero is to be determined to get complete transfer function. This falls in line with the equivalence of 'Brahma' (ब्रह्म) with 'Dahara' (दहर), as prescribed in Chāndogya - अस्मिन्ब्रह्मपुरे दहरं पुंडरीकं वेश्म दहरोऽस्मिन। छां.८.१.१. The complete transfer is obtained as follows:
Let impulse response be,

$$h[n] = \underset{\uparrow}{\left[h_0\, h_1\, h_2\, h_3\right]}$$

So $h[n]$ is orthogonal to its even shifts *e.g.* shifts of 2,4,6 etc. So the only non-trivial relation is obtained by the dot product of $h[n]$ and $h[n-2]$.

$$h[n] = \underset{\uparrow}{\left[h_0\, h_1\, h_2\, h_3\right]}$$

$$h[n-2] = \underset{\uparrow_{n=2}}{\left[\ldots\ldots h_0\, h_1\, h_2\, h_3\right]}$$

Hence their dot product is $(h_0h_2+h_1h_3)$ and because of orthogonality of dot product with respect to even shifts,

$$h_0h_2+h_1h_3=0 \tag{4.19}$$

Now system function can be expressed as,

$$H_0(z) = h_0+h_1z^{-1}+h_2z^{-2}+h_3z^{-3} \tag{4.20}$$

Also as $H_0(z)$ has a factor of $(1+z^{-1})^2$, so in general $H_0(z)$ can be written as,

$$H_0(z) = C_0(1+z^{-1})^2(1+B_0z^{-1})$$

where C_0 is a constant.
Here two zeros are constrained at $z=-1$. If we neglect the constant for time being and then

expanding the previous expression, we can write,

$$\begin{aligned} H_0(z) &= 1+2z^{-1}+z^{-2}+B_0z^{-1}+2B_0z^{-2}+B_0z^{-3} \\ H_0(z) &= 1+(2+B_0)z^{-}1+(1+2B_0)z^{-2}+B_0z^{-3} \end{aligned} \tag{4.21}$$

Comparing the coefficients of powers of z^{-1} from equation 4.20 and 4.21, we get,

$$\begin{aligned} h_0 &= 1 \\ h_1 &= 2+B_0 \\ h_2 &= 1+2B_0 \\ h_3 &= B_0 \end{aligned}$$

Putting this value in equation (1),we get,

$$(1+2B_0)+(2+B_0)B_0 = 0$$

$$\Rightarrow B_0^2+4B_0+1 = 0$$

Solving the above quadratic equation, we get,

$$B_0 = (-4\pm 2\sqrt{3})/2 = -2\pm\sqrt{3} \tag{4.22}$$

Now the implication of B_0 is that the third zero of $H_0(z)$ is at B_0. For $B_0 = -2-\sqrt{3}$ *i.e.* $B_0 = -3.732$, the zero is outside the unit circle in the z-plane. Because $|B_0| > 1$, this will not become the minimum phase implementation. The magnitude response being the same as required. So there will be more phase delay and group delay in the system which is not desired. But for $B_0 = -2+\sqrt{3}$ *i.e.* $B_0 = -0.268$, the zero is inside the unit circle in the z-plane and so the system remains the minimum phase system, because $|B_0| < 1$. So we choose $B_0 = -2+\sqrt{3}$ *i.e.* $B_0 = -0.268$.
So the impulse response of the analysis side low pass filter of length 4 of Daubechies family is, shown in table (4.3) below.

In this derivation process we have neglected the constant C_0. To find C_0, let's recall:

$$\kappa_0(z)+\kappa_0(-z) = constant \tag{4.23}$$

where,

$$\kappa_0(z) = H_0(z)H_0(-z^{-1}) \tag{4.24}$$

In order to choose the constant C_0 the easy thing to do is to make the norm of the impulse response of $H_0(z)$ unity in the sense of l_2 norm. Now the dot product of a sequence with itself gives the

square of its l_2 norm. So the sequence corresponding to $\kappa_0(z)$ at the 0^{th} location is essentially the squared norm in the $l_2(Z)$ of $[h_0\ h_1\ h_2\ h_3]$. So C_0 chosen such as,

$$\begin{aligned} C_0^2(h_0^2+h_1^2+h_2^2+h_3^2) &= 1 \\ C_0^2 = 1/4.287 &= 0.233 \\ C_0 &= 0.4829 \end{aligned} \tag{4.25}$$

h_0	h_1	h_2	h_3
0.4829*1=0.4829	0.4829*1.732=0.8364	0.4829*0.464=0.2241	0.4829*(-0.268)=-0.129

Table 4.3: Normalized Impulse Response

4.12.1 Calculation of Scaling Function

Now our next job is to calculate $\phi(t)$ and $\psi(t)$ from the calculated impulse response. To calculate the scaling function $\phi(t)$ we have to keep on compressing and convolving $h[n]$ iteratively. This is done as follows:

Let us treat $h[n]$ as the set of coefficients of an impulse train containing only 4 impulses in the continuous time domain such as the function in the continuous time domain is $h(t)$ such as,

$$h(t) = h_0\delta(t) + h_1\delta(t-T) + h_2\delta(t-2T) + h_3\delta(t-3T) \tag{4.26}$$

These calculations are well supported by several clear descriptions in Advaita Vedānta Philosophy. For examples of which are as follows - इदं सर्वं यदयमात्मा। बृ.२-४-६, आत्मैवेदं सर्वम्। छां.७-२५-२, ब्रह्मैवेदममृतं पुरस्तात। मुं.२-२-११, सर्वं खल्विदं ब्रह्म। छां.३-१४-१

Accordingly the shape of $h(2t), h(4t), h(8t)$ *etc.* are shown in figure 14.2.

The iterative convolutional kernel re-confirms the doctrine stated in Chāndogya Upaniṣada (छान्दोग्य उपनिषद) - यथा सोम्य, एकेन मृत्पिण्डेन सर्वं मृण्मयं विज्ञातं स्यात् वाचारम्भणं विकारो नामधेयं मृत्तिकेत्येव सत्यम्। छां.६-१-४.

For the iterative convolution, first $h(t)$ is convolved with $h(2t)$ and the result is convolved with $h(4t)$. Then the result is convolved with $h(8t)$ and so on. If this convolution process with compressed version of $h(t)$ is carried on infinitely, we will get the scaling function $\phi(t)$.

There is an interesting conclusion of this iterative convolution process. Suppose $h(t)$ has length L *i.e.* $h(t) = 0$ for any $t < 0$ and for any $t > L$. So $h(2t)$ has a length $L/2$, $h(4t)$ has a length $L/4$, $h(8t)$ has a length $L/8$ and so on. Now the convolution of $h(t)$ with $h(2t)$ gives result with length $L + L/2$. This result when convolved with $h(4t)$, gives a result with length $L + L/2 + L/4$. So going on this way we can get $\phi(t)$, which is of length $L + L/2 + L/4 + ... = 2L$.

It means $\phi(t)$ is zero for any $t < 0$ and $t > 2L$ *i.e.* we converge towards a compactly supported scaling function. The independent variable region over which the scaling function is non-zero is finite. This

Table 4.4:

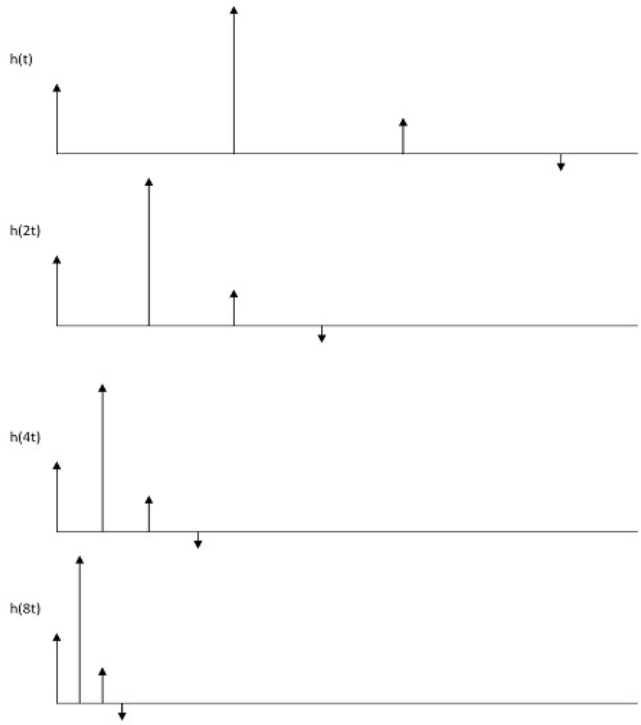


Figure 4.26: Shape of $h(t), h(2t), h(4t)$ and $h(8t)$

is the most important contribution made by Daubechies. Before Daubechies came up with these set of filter banks idea of neatly constructing a family of compactly supported multiresolution analysis was not existing in the literature. So this is a very useful contribution in MRA.

Interpretation for Daubechies filter banks

The theories of wavelets and filter banks developed in parallel. Using filter banks effectively to generate compactly supported scaling function is an important contribution by Daubechies.
A different interpretation can be thought about the Daubechies filter banks. The high pass analysis filter bank essentially reduces the degree of a polynomial input. Suppose there is an input of the form $x[n] = a + bn$, where a and b are constants. So the factor $(1 - z^{-1})$ in the high pass filter reduces the degree of the input. If there had been only this term in the high pass filter, the output would have been in the form of $y[n] = a + bn - a - b(n-1) = b$. If there had been another term of $(1 - z^{-1})$ the output becomes 0. So in Daubechies length 4 (abbreviated as Daub-4) high pass filter bank the polynomial is annihilated. On the other side, in the low pass analysis filter, because of the $(1 + z^{-1})$ term, the output becomes $y[n] = a + bn + a + b(n-1) = 2a + 2bn - b$. It can similarly be extended for another $(1 + z^{-1})$ term. This means that the polynomial form of expression remains in the low pass branch and the high pass branch contains some residual component, thereby retaining a few more smoother terms related to polynomial in the low pass branch and removing them from the high pass branch.
While calculating the iterative convolution we saw that the scaling function thus obtained has a compact support. But it is important to note that had it been taken any arbitrary values of

h_0, h_1, h_2, h_3 the iterative convolution process might not have converged to a function with finite number of discontinuities. But beauty of Daubechies family is that whatever be the filter length, the convolution always converges to a function with finite number of discontinuities. The specialty that makes the convolution to converge is denoted by a term 'regularity' in Wavelet literature, *i.e.* the filters need to obey regularity for the iterative convolution to converge. This regularity comes because of the presence of the zeros in the system function. One guaranteed way of forcing regularity is to introduce factors of $(1+z^{-1})$, *i.e.* adding zeroes at $z=-1$ *i.e.* $\omega=\pi$ in the low pass analysis filters. In case of high pass analysis filter the zeros are added at $z=1$ *i.e.* $\omega=0$. So the zeros are put at the extreme high frequency in low pass filter and at the extreme low frequency in high pass filter. In case of different filter banks, the number of zeros are as listed in table (4.5).

Table 4.5: Daunechies family member and corresponding zeros

Haar	1 zeros
Daub-4	2 zeros
Daub-6	3 zeros

Higher is the filter length more regular is the Daubechies filter. This means the function to which we converge by iterative convolution becomes more and more smooth *i.e.* they have more and more derivative which are continuous. In Daubechies-4 there are some issues in the differentiability but in the higher order filters that is also taken care of.

Next Daubechies family member Daub-6

The next member of Daubechies family is a length 6 filter of degree 5. So in that case $H_0(z)$ can be written as,

$$H_0(z) = C_0(1+z^{-1})^3(1+\widetilde{B_0}z^{-1})(1+\widetilde{B_1}z^{-1}) \tag{4.27}$$

here C_0 is a constant. Three zeros are constrained and two are free ($\widetilde{B_0}$ and $\widetilde{B_1}$). Let the impulse response be,

$$h[n] = \underset{\uparrow}{\left[h_0\, h_1\, h_2\, h_3\, h_4\, h_5\right]}$$

so $h[n]$ is orthogonal to its even shifts *e.g.* shift of 2, 4, 6 etc. So the non trivial relations are obtained by the dot product between $h[n]$ and $h[n-2]$ or $h[n-4]$. Here,

$$h[n] = \underset{\uparrow}{\left[h_0\, h_1\, h_2\, h_3\, h_4\, h_5\right]}$$

$$h[n-2] = \underset{\uparrow_{n=2}}{\left[\ldots\ldots h_0\, h_1\, h_2\, h_3\, h_4\, h_5\right]}$$

$$h[n-4] = \underset{\uparrow_{n=4}}{\left[\ldots\ldots\ldots\ldots\ldots h_0\, h_1\, h_2\, h_3\, h_4\, h_5\right]}$$

$$\Rightarrow h_2h_0 + h_3h_1 + h_4h_2 + h_5h_3 = 0 \tag{4.28}$$

$$\Rightarrow h_4h_0 + h_5h_1 = 0 \tag{4.29}$$

From equations 4.27, 4.29 and 4.29, we can find out the values of $\widetilde{B_0}$ and $\widetilde{B_1}$. There from we can construct the impulse response in a way similar to Daub-4 case.
This type of filter banks are called Conjugate Quadrature filter bank. The reason for this nomenclature is that the low pass and the high pass filter frequency responses are π apart from each other. So the principal equation governing the conjugate quadrature filter is,

$$\kappa_0(z) + \kappa_0(-z) = constant$$

where,

$$\kappa_0(z) = H_0(z)H_0(z^{-1})$$

$$\kappa_0(e^{j\omega}) = H_0(e^{j\omega})H_0(e^{j\omega})$$

If the frequency response is real then,

$$\left|H_0(e^{j\omega})\right|^2 = \kappa_0(e^{j\omega}) \tag{4.30}$$

This means designing a conjugate quadrature filter bank is essentially designing $\kappa_0(e^{j\omega})$ only. $\kappa_0(z)$ corresponds to a real and even impulse response with the constraints that even samples of the impulse response are all '0' except at the 0th sample.
There are many ways to design such filter banks. Once we have $\kappa_0(z)$ we find its roots. For each root there are pairs of reciprocal roots $H_0(z)$ and $H_0(z^{-1})$. Out of each reciprocal root pair, one root is assigned to $H_0(z)$ and the other automatically gets assigned to $H_0(z^{-1})$. The Daubechies filters are one class of conjugate quadrature filters.
Our future study will aim at what we are looking for out of these filter banks in both time and frequency domains.

4.13 Daub-4 and Daub-6 Design Details

This aims at using linear algebra and matrices.
we wish to come up with design procedure to construct 'even'length Doubechies filter.

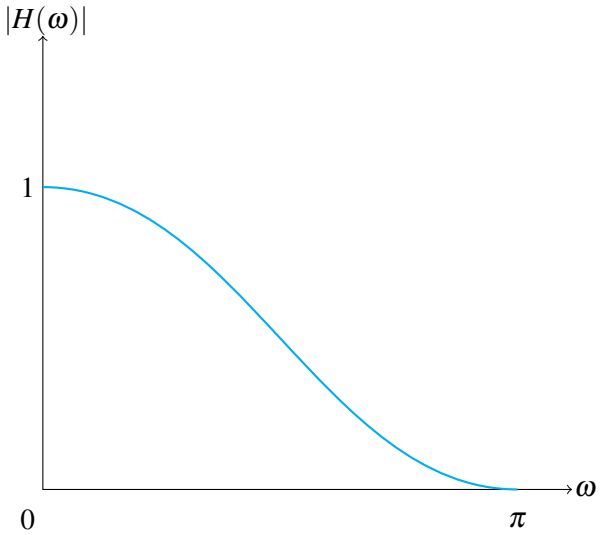
Let
$h = \{h_0, h_1, \cdots h_L\}$ be the LPF coefficients and
$g = \{g_0, g_1, \cdots g_L\}$ be the HPF coefficients and

Let both these filters be finite impulse response filters
e.g. if , $h = \{h_0, h_1, h_2\} = \left\{\frac{1}{4}, \frac{1}{2}, \frac{1}{4}\right\}$

Then the Fourier series $h(\omega)$will be

$$H(\omega) = \left(\frac{1}{4} + \frac{1}{2}\cdot e^{j\omega} + \frac{1}{4}\cdot e^{2j\omega}\right) = e^{j\omega}\cdot\frac{1}{2}(1+\cos\omega) \tag{4.31}$$

If we plot the magnitude graph over $[0,\pi]$,then



$\therefore$ low pass filter conditions could be

$H|(0)| = \frac{1}{2}(1+\cos 0) = 1$ &

$H|(\pi)| = \frac{1}{2}(1+cos\pi) = 0$

Now,

$$H(\omega) = \sum_{k=0}^{L} h_k \cdot e^{-jk\omega} \tag{4.32}$$

Now if $g = \{g_0, g_1, g_2, \cdots g_L\}$

$$G(\omega) = \sum_{k=0}^{L} g_k \cdot e^{-jk\omega} \tag{4.33}$$

High-pass conditions will be $|G(0)| = 0$ & $|G(\pi)| = 1$

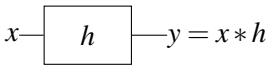
To design Daub-4 & Daub-6,

L=3 & 5.

Now,let's construct system of linear & quadratic equations that coefficient of 'h' show satisfy.

Design 1- Daubechies-4
h=$\{h_0, h_1, h_2, h_3\}$
g=$\{g_0, g_1, g_2, g_3\}$

Typical output of any filtered is mathematically captured by 'convolution'.



$$y_n = \sum_{k=0}^{L} h_k \cdot x_{n-k} = h_0 \cdot x_n + h_1 \cdot x_n + \cdots \tag{4.34}$$

$\therefore$ For causal system & sequences (i.e. no $y\_1, y\_2, \cdots and x\_1, x\_2, \cdots$ terms)

$$\begin{bmatrix} y_0 \\ y_1 \\ \vdots \\ y_2 \end{bmatrix} = H \cdot \begin{bmatrix} x_0 \\ x_1 \\ \vdots \\ x_2 \end{bmatrix} \tag{4.35}$$

Important things to note about 'H' matrix:

1. Since it represents system in matrix form it is a system of linear equations.
2. 'ho's constitute main diagonal.
3. With reference to main diagonal,upper triangle is '0',which suggests, system is causal $[H_{ij} = 0 \forall j > i, i, j \in Z]$
4. Since all diagonals are constant,it indicates,system is shift invariant.
5. $\therefore$ 'H' is a causal LTI system.

Similarly,for high pass filter 'G' matrix can be built.'H' and 'G' together constitute wavelet matrix 'w',as in wavelets we use filter bank,combination of LPF(H) and HPF(G).
E.g. if in case of Haar,
h[n]=$\left\{\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right\}$ and g[n]=$\left\{\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right\}$
'W' matrix will be,

$$W = \begin{bmatrix} H \\ G \end{bmatrix}$$

,& look like

$$W = \sqrt{2} \cdot \begin{bmatrix} 12 & 12 & 0 & 0 \\ 0 & 0 & 12 & 12 \\ -12 & 12 & 0 & 0 \\ 0 & 0 & -12 & 12 \end{bmatrix} \tag{4.36}$$

This is Daub-2 or Haar.
Now,for Daub-4,the filter length will be '4' & W matrix will be 8×8.
Let's call that as W_{D4}.

$$W_{D4} = \begin{bmatrix} h_3 & h_2 & h_1 & h_0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_3 & h_2 & h_1 & h_0 & 0 & 0 \\ 0 & 0 & 0 & 0 & h_3 & h_2 & h_1 & h_0 \\ h_1 & h_0 & 0 & 0 & 0 & 0 & h_3 & h_2 \\ - & - & - & - & - & - & - & - \\ g_3 & g_2 & g_1 & g_0 & 0 & 0 & 0 & 0 \\ 0 & 0 & g_3 & g_2 & g_1 & g_0 & 0 & 0 \\ 0 & 0 & 0 & 0 & g_3 & g_2 & g_1 & g_0 \\ g_1 & g_0 & 0 & 0 & 0 & 0 & g_3 & g_2 \end{bmatrix} = \begin{bmatrix} H_{D4} \\ ------ \\ G_{D4} \end{bmatrix} \tag{4.37}$$

Now,for W_{D4} to become a transformation matrix.

1. The inverse should exist.
2. W_{D4} should be unitary,i.e. for real values,$W_{D4}^{-1} = W_{D4}^{T}$.
3. W_{D4} should be orthogonal.

Assuming 1^{st} two criteria,
For the orthogonality,

$$W_{D4} \cdot W_{D4}^{T} = I$$

$$\therefore W_{D4} \cdot W_{D4}^{T} = \begin{bmatrix} H_{D4} \\ G_{D4} \end{bmatrix} \begin{bmatrix} H_{D4} & G_{D4} \end{bmatrix} = \begin{bmatrix} H_{D4} \cdot H_{D4}^{T} & H_{D4} \cdot G_{D4}^{T} \\ G_{D4} \cdot H_{D4}^{T} & G_{D4} \cdot G_{D4}^{T} \end{bmatrix} = \begin{bmatrix} I_4 & 0_4 \\ 0_4 & I_4 \end{bmatrix}$$

$$\therefore H_{D4} \cdot H_{D4}^{T} = I_4$$

$$\therefore \begin{bmatrix} h_3 & h_2 & h_1 & h_0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_3 & h_2 & h_1 & h_0 & 0 & 0 \\ 0 & 0 & 0 & 0 & h_3 & h_2 & h_1 & h_0 \\ h_1 & h_0 & 0 & 0 & 0 & 0 & h_3 & h_2 \end{bmatrix} \cdot \begin{bmatrix} h_3 & 0 & 0 & h_1 \\ h_2 & 0 & 0 & h_0 \\ h_1 & h_3 & 0 & 0 \\ h_0 & h_2 & 0 & 0g_0 \\ 0 & h_1 & h_3 & 0 \\ 0 & h_0 & h_2 & 0 \\ 0 & 0 & h_1 & h_3 \\ 0 & 0 & h_0 & h_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Let's work on 1^{st} row,1^{st} column $\rightarrow$ should lead to '1'
& 1^{st} row,2^{nd} column $\rightarrow$ should lead to '0'

$\therefore$

$$h_3^2 + h_2^2 + h_1^2 + h_0^2 = 1 \tag{4.38}$$

&

$$h_1 \cdot h_3 + h_0 \cdot h_2 = 0 \tag{4.39}$$

The orthogonality between 'h' & 'g' plays vital role.One way of understanding orthogonality is 'zero' dot product between two vectors.'Dot' product is element by element product.For every 'h', if we create corresponding 'g' by flipping the sequence & making alternate samples go negative, then that will certainly achieve orthogonalization.
e.g. if $h = \{1,2,3,4\}$
then g=$\{4,-3,2,-1\}$ is a good candidate to produce orthogonal framework.
$\langle h \cdot \rangle = [1 \times 4 + 2 \times (-3) + 3 \times (2) + 4 \times (-1)] = 0$!
Mathematically,this can be captured using:
$g_k = (-1)^k \cdot h_{L-k}$,
where, h=$\{h_0, h_1, \cdots, hL\}$ &$g = \{g_0, g_1, \cdots, g_L\}$
This grantees $\langle h, g \rangle = [h \cdot g] = 0$
$\therefore g = \{g_0, g_1, g_2, g_3\} = \{h_3, -h_2, h_1, -h_0\}$
Now,

$$H_{D4} \cdot G_{D4}^T = [0]_{4\times 4}$$

$$\therefore \begin{bmatrix} h_3 & h_2 & h_1 & h_0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_3 & h_2 & h_1 & h_0 & 0 & 0 \\ 0 & 0 & 0 & 0 & h_3 & h_2 & h_1 & h_0 \\ h_1 & h_0 & 0 & 0 & 0 & 0 & h_3 & h_2 \end{bmatrix} \cdot \begin{bmatrix} -h_0 & 0 & 0 & -h_2 \\ h_1 & 0 & 0 & h_3 \\ -h_2 & -h_0 & 0 & 0 \\ h_3 & h_1 & 0 & 0 \\ 0 & -h_2 & -h_0 & 0 \\ 0 & h_3 & h_1 & 0 \\ 0 & 0 & -h_2 & -h_1 \\ 0 & 0 & h_3 & h_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Similarly,
$G_{D4} \cdot H_{D4}^T = [0]_{4\times 4}$

$$\begin{bmatrix} -h_0 & h_1 & -h_2 & h_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & -h_0 & h_1 & -h_2 & h_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & -h_0 & h_1 & -h_2 & h_3 \\ -h_2 & h_3 & 0 & 0 & 0 & 0 & -h_1 & h_2 \end{bmatrix} \cdot \begin{bmatrix} h_3 & 0 & 0 & h_1 \\ h_2 & 0 & 0 & h_0 \\ h_1 & h_3 & 0 & 0 \\ h_0 & h_2 & 0 & 0g_0 \\ 0 & h_1 & h_3 & 0 \\ 0 & h_0 & h_2 & 0 \\ 0 & 0 & h_1 & h_3 \\ 0 & 0 & h_0 & h_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Also,$G_{D4} \cdot G_{D4}^{'} = I_4$

$$\therefore \begin{bmatrix} -h_0 & h_1 & -h_2 & h_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & -h_0 & h_1 & -h_2 & h_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & -h_0 & h_1 & -h_2 & h_3 \\ -h_2 & h_3 & 0 & 0 & 0 & 0 & -h_1 & h_2 \end{bmatrix} \cdot \begin{bmatrix} -h_0 & 0 & 0 & -h_2 \\ h_1 & 0 & 0 & h_3 \\ -h_2 & -h_0 & 0 & 0 \\ h_3 & h_1 & 0 & 0 \\ 0 & -h_2 & -h_0 & 0 \\ 0 & h_3 & h_1 & 0 \\ 0 & 0 & -h_2 & -h_1 \\ 0 & 0 & h_3 & h_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4.40}$$

We again confirm,

$$h_0^2 + h_1^2 + h_2^2 + h_3^2 = 1$$
$$h_0 h_2 + h_1 h_3 = 0$$

Now,
let's impose conditions so that 'h' is truly a low pass filter on [0,π]
Being a LPF,it should pass frequencies at $\omega = 0$ & block frequencies at $\omega = \pi$.
Let $H(\omega)$ be the Fourier representation of h(n)in the form of series,

$$H(\omega) = h_0 + h_1 \cdot e^{j\omega} + h_2 \cdot e^{2\cdot j\omega} + h_3 \cdot e^{3\cdot j\omega} \tag{4.41}$$

Let's impose first condition to pass frequencies at w=0,and let the magnitude by 'Unit' decided by us.

$$\therefore H(\omega, 0) = h_0 + h_1 \cdot e^0 + h_2 \cdot e^0 + h_3 \cdot e^0 = Unit \tag{4.42}$$

What unit should we choose.Well,we choose it carefully to normalize orthogonal system,thus making it orthonormal.
Recall the MRA framework we saw,h_k is linked with $\phi(\cdot)$ and s_k is linked with $\psi(\cdot)$.For set

$\left\{2^{\frac{j}{2}}\phi(2^j x-k)\right\}$,being orthonormal & linearly independent,it constitutes a basis for vector space V^j at window of analysis $W_a = 12^j$.
This is truly orthonormal on $(-\infty,\infty)$,since

$$\int_{-\infty}^{\infty} 2^{\frac{j}{2}}\phi(2^j x-k)\cdot\phi(2^j x-l)dx = \begin{cases}0, & k\neq l\\ 1, & k=l\end{cases} \tag{4.43}$$

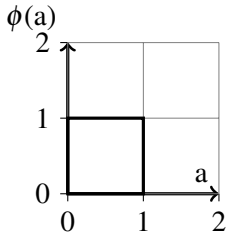
This is condition for orthogonality,whole makes it orthonormal is normalizing factor of $2^{\frac{j}{2}}$ in $\left\{2^{\frac{j}{2}}\phi(2^j x-k)\right\}$.
If we take norm of $\phi(2^j x-k)$,we get that factor.

$$\|\phi(2^j x-k)\|^2 = \int_{-\infty}^{\infty}\phi^2(2^j x-k)dx \qquad = \frac{1}{2^j}\int_{-\infty}^{\infty}\phi^2(2^j x-k)\cdot 2^j dx \tag{4.44}$$

Let's put $2^j x-k=a$
Differentiating both sides
$2^j\cdot dx-0=da$
$\therefore 2^j dx=da$

$$\|\phi(2^j x-k)\|^2 = \frac{1}{2^j}\int_{-\infty}^{\infty}\phi^2(a)dx \tag{4.45}$$

Let's prove it for Haar,where



$\|\phi(2^j x-k)\|^2 = \frac{1}{2^j}\int_0^1(1)^2 da = \frac{1}{2^j}(1) = \frac{1}{2^j}$

Taking square root for positives,
$\|\phi(2^j x-k)\|^2 = \frac{1}{2^j}$

$\therefore$ We can divide $\phi(2^j x-k)$ by its norm to get orthogonal basis converted to orthonormal basis,
$\frac{\phi(2^j x-k)}{\frac{1}{j^{\frac{j}{2}}}} = 2^{\frac{j}{2}}x-k) = 2^{\frac{j}{2}}\phi(2^j x-k)$
From the point of view of nested subspaces, we have to account for $\sqrt{2}$ every time we move from one subspace to another.

$$\cdots \quad \overset{\sqrt{2}}{\frown} \quad \overset{\sqrt{2}}{\frown} \quad \overset{\sqrt{2}}{\frown} \quad \overset{\sqrt{2}}{\frown} \quad \cdots$$
$$\cdots \quad V_{-2} \subset V_{-1} \subset V_0 \subset V_1 \subset V_2 \quad \cdots$$

This is a 'dyadic' style of realizing discrete wavelet filters.
$\therefore$ We choose 'Unit' to be '$\sqrt{2}$' to maintain orthogonality.
We will put this in equation (4.41),

$$h_0 + h_1 + h_2 + h_3 = \pm\sqrt{2} \tag{4.46}$$

We will use this as a final check whether the LPF coefficient satisfy this condition or not.
Let's impose second low pass condition of complete attenuation at w=π.
$\therefore H(\pi) = 0 = h_0 + h_1 \cdot e^{j\pi} + h_2 \cdot e^{2j\pi} + h_3 \cdot e^{3j\pi}$
Using euler's identity $e^{j\pi}$=-1
$e^2 j\pi = (e^j\pi)^2 = (-1)^2 = 1$
$e^3 j\pi = (e^j\pi)^3 = (-1)^3 = -1$

$$H(\pi) = 0 = h_0 - h_1 + h_2 - h_3 \tag{4.47}$$

'g_k'being a highpass filter,let's impose conditions accordingly,

$$g_k = \left\{ \underset{\uparrow}{g_0, g_1, g_2, g_3} \right\}$$

$\therefore G(\omega) = g_0 + g_1 \cdot e^{j\omega} + g_2 \cdot e^{2j\omega} + g_3 \cdot e^{3j\omega}$
Put $\{g_0, g_1, g_2, g_3\} = \{{}_+h_3, {}^-h_2, {}_+h_1, {}^-h_0\}$

$\therefore G(\omega) = h_3 - h_2 \cdot e^{j\omega} + h_1 \cdot e^{2j\omega} - h.e^{3j\omega}$
G(0)=0 -(block low frequencies)

$$\therefore h_3 - h_2 + h_1 - h_0 = 0 \tag{4.48}$$

$G(\pi)$ =unit= $\sqrt{2}$
$|h_0 + h_1 + h_2 + h_3| = \sqrt{2}$ (another check condition)

Let's combine two orthogonality conditions and one

LPF condition towards finding h_k From (1),(2)& (5)

$$h_0^2 + h_1^2 + h_2^2 + h_3^2 = 1$$
$$h_0 h_2 + h_1 h_3 = 0 \quad \text{(D1)}$$
$$h_0 - h_1 + h_2 - h_3 = 0$$

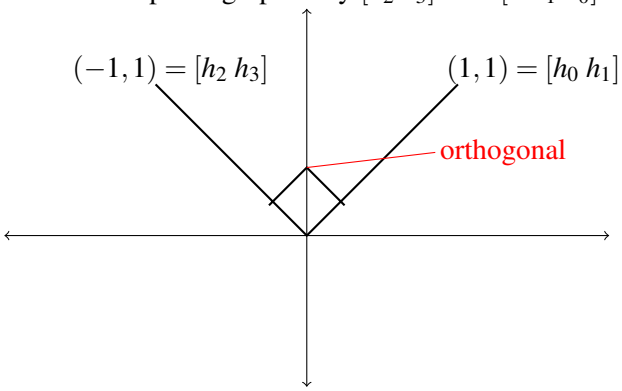
These are our design Equations!
let's start with the following equation
$h_0 + h_2 h_1 h_3 = 0$

This clearly implies $[h_0\ h_1]^T$ and$[h_2 \quad h_3]^T$ are orthogonal
i.e. inner or dot product$\langle [h_0\ h_1], [h_2\ h_3] \rangle$ suggests
$h_0 h_2 + h_1 + h_3 \longrightarrow$ going to '0'

e.g. $h_0 = 1$,$h_1 = 1$,$h_2 = -1$,$h3 = 1$ ensures

$[h_o\ h_1]^T \perp [h_2\ h_3]^T$

We can also prove graphically $[h_2\ h_3]^T \perp c[-h_1\ h_0]^T$



$\therefore$ it is easy to see that

$$[h_2\ h_3]^T \perp c[-h_1\ h_0]^T \tag{4.49}$$

with c=1 in this case!

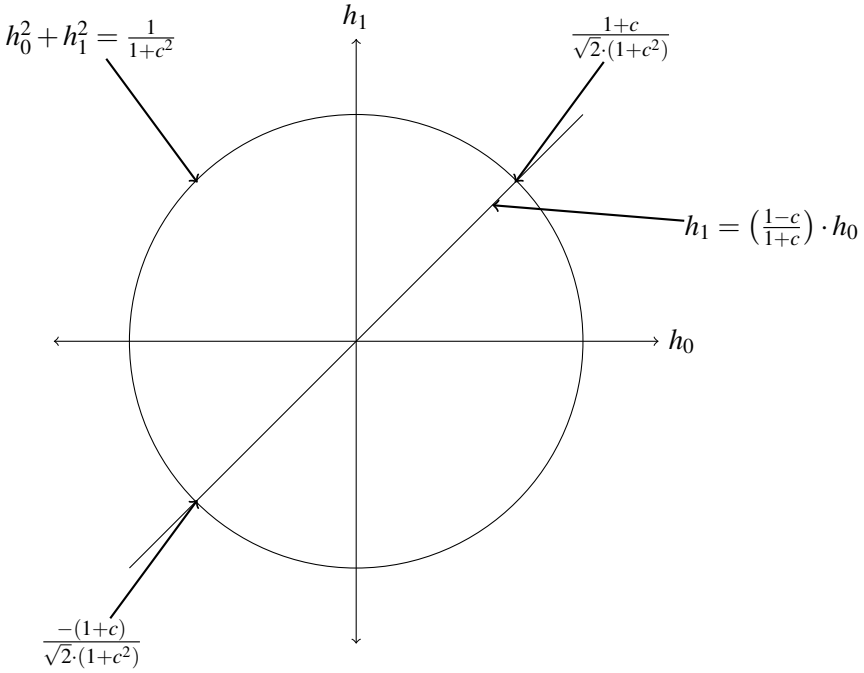
For c$\neq$ 0,if we insert
$[h_2 h_3]^T \perp c\,[-h_1 h_0]^T$ in equation 4.39
We get

$$h_0^2 + h_1^2 = \frac{1}{1+c^2} \tag{4.50}$$

One way of looking at equation (**??**) is
$h_2 = -ch_1 \& h_3 = ch_0$
Let's plug these values in (4.47)
$\therefore h_0 - h_1 - ch_1 + ch_0$=0
$\therefore h_0(1-c) + h_1(1+c) = 0$

$$h1 = \left(\frac{1-c}{1+c}\right) h_0 = \begin{cases} \quad c \neq l \end{cases} \tag{4.51}$$

$\therefore$ (4.50) is clearly a equation of circle of radius $\frac{1}{\sqrt{1+c^2}}$ & (4.51) is a equation of straight line with slope $\frac{1-c}{1+c}$
Graphically,



$\therefore$ For any given c$\neq$-1,the line with slope of $\frac{1-c}{1+c}$ intersects circle of radius $\frac{1}{\sqrt{1+c^2}}$twice

$\therefore$ For each 'c' there are 2 possible solutions,that will satisfy design conditions D_1.
Let's simplify

$$h_0^2+h_1^2=\frac{1}{1+c^2}$$

$$h_0^2+\left(\frac{1-c}{1+c}\right)^2\cdot h_0^2=\frac{1}{1+c^2}$$

$$h_0^2\left(1+\left(\frac{1-c}{1+c}\right)^2\right)=\frac{1}{1+c^2}$$

$$h_0^2\left[\left(\frac{(1+c)^+(1-c)^2}{(1+c)^2}\right)\right]=\frac{1}{1+c^2}$$

$$h_0^2\left[\left(\frac{(1+2c+c^2+1-2c+c^2}{(1+c)^2}\right)\right]=h_0^2\left[\frac{2(1+c^2)}{1+c^2}\right]$$

$$\therefore \quad 2h_0^2=\frac{1}{1+c^2}\cdot\frac{(1+c)^2}{1+c^2}$$

$$\therefore \quad h_0^2=\frac{1+c^2}{2(1+c^2)^2}$$

$$\therefore \quad h_0=\pm\frac{1+c}{\sqrt{2}(1+c^2)} \longleftrightarrow \text{the 'two' solutions!}$$

Let's choose (+ve)root

$$h_0 = +\frac{1+c}{\sqrt{2}(1+c^2)} \tag{4.52}$$

By plug & solve

$$h_0 = \frac{1-c}{\sqrt{2}(1+c^2)}\,, h_2 = +\frac{-c(1-c)}{\sqrt{2}(1+c^2)}\,, h_3 = +\frac{-c(1+c)}{\sqrt{2}(1+c^2)}$$

D1 is a system of three equations & we are trying to figure out four unknowns
Let's add one more LP condition
$H(\pi) = 0$
As ideal filtering not possible we impose
$H'(\pi) = 0$
First order difference (derivative) will give us,

$$H"(\omega)|_{\omega=\pi} = jh_1e^{j\omega} + 2jh_2e^{2j\omega} + 3jh_3e^{3j\omega}|_{\omega=\pi}$$

$$0 = j\left(h_1(-1) + 2h_2(-1)^2 + 3h_3(-1)^3)\right)$$

$$\therefore h_1 - 2h_2 + 3h_3 = 0 \tag{4.53}$$

Let's add (4.53) in D1 to get complete set D2

$$\left.\begin{array}{c} h_0^2 + h_1^2 + h_2^2 + h_3^2 = 1 \\ h_0 + h_2h_1h_3 = 0 \\ h_0 - h_1 + h_2 - h_3 = 0 \\ h_1 - 2h_2 + 3h_3 = 0 \end{array}\right\} \text{(D2)}$$

Now,Let's plug $h_2 = -ch_1$ & $h_3 = ch_0 in (4.53)$

$$h_1 + 2ch_1 + ch_0 = h_1(1+2c) + 3ch_0 = 0$$

$$\therefore\ h_1 = -\left(\frac{3c}{(1+2c)\cdot h_0}\right)$$

we already know the slope matches with $\frac{1-c}{1+c}$

$$\therefore\ \frac{-3c}{1+2c} == \frac{1-c}{1+c}$$

$$\therefore\ -3c - 3c^2 = 1 + 2c - c - 2c^2$$

$\therefore\ -3c-3c^2 = 1+c-2c^2$

$$\therefore\ c^2+4c+1=0 \tag{4.54}$$

Roots of (4.54) wiil be

$c = -2 \pm \sqrt{3}$

$c = -2-\sqrt{3}$ makes system slow& sluggish as the zero lies outside unit circle& it ensure max-phase system.
$\therefore$ we choose minimum phase $c = -2+\sqrt{3}$ solution by backward substitution in D2

$$h_0 = \frac{1+\sqrt{3}}{4\sqrt{2}} \qquad h_1 = \frac{3+\sqrt{3}}{4\sqrt{2}}$$

$$h_3 = \frac{1-\sqrt{3}}{4\sqrt{2}} \qquad h_2 = \frac{3-\sqrt{3}}{4\sqrt{2}}$$

Now, $g_k = (-1)^k h_3 - k,\ k = 0,1,2,3$

$$g_0 = h_3 = \frac{1-\sqrt{3}}{4\sqrt{2}} \qquad g_1 = -h_2 = -\frac{3-\sqrt{3}}{4\sqrt{2}}$$

$$g_2 = h_1 = \frac{3+\sqrt{3}}{4\sqrt{2}} \qquad g_3 = -h_0 - \frac{1+\sqrt{3}}{4\sqrt{2}}$$

Now let,s repeat this for Daub-6 where

$h = \{h_0 \cdots h_5\}$
$g = \{g_0 \cdots g_5\}$

$$W_{10} = \begin{bmatrix} h_5 & h_4 & h_3 & h_2 & h_1 & h_0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_5 & h_4 & h_3 & h_2 & h_1 & h_0 & 0 & 0 \\ 0 & 0 & 0 & 0 & h_5 & h_4 & h_3 & h_2 & h_1 & h_0 \\ h_1 & h_0 & 0 & 0 & 0 & 0 & h_5 & h_4 & h_3 & h_2 \\ h_3 & h_2 & h_1 & h_0 & 0 & 0 & 0 & 0 & h_5 & h_4 \\ - & - & - & - & - & - & - & - & - & - \\ g_5 & g_4 & g_3 & g_2 & g_1 & g_0 & 0 & 0 & 0 & 0 \\ 0 & 0 & g_5 & g_4 & g_3 & g_2 & g_1 & g_0 & 0 & 0 \\ 0 & 0 & 0 & 0 & g_5 & g_4 & g_3 & g_2 & g_1 & g_0 \\ g_1 & g_0 & 0 & 0 & 0 & 0 & g_5 & g_4 & g_3 & g_2 \\ g_3 & g_2 & g_1 & g_0 & 0 & 0 & 0 & 0 & g_5 & g_4 \end{bmatrix} = \begin{bmatrix} H_{D4} \\ - - - \\ G_{D4} \end{bmatrix}$$

We can extend D1 conditions directly

$$\begin{aligned} h_0^2 + h_1^2 + h_2^2 + h_3^2 + h_4^2 + h_5^2 &= 1 \\ h_0 h_2 + h_1 h_3 + h_2 h_4 + h_3 h_5 &= 0 \\ h_0 h_4 + h_1 h_5 &= 0 \end{aligned} \tag{D3}$$

$g = h_5, h_4, h_3, h_2, h_1, h_0$

$\therefore\ H_{D10} \cdot {H_{D10}}^T = G_{D10} \cdot {G_{D10}}^T = I_5\ \ \&$

$\therefore\ H_{D10} \cdot {G_{D10}}^T = G_{D10} \cdot {H_{D10}}^T = 0_5$

Low pass conditions-

$H(\pi) = 0 = h_0 - h_1 + h_2 - h_3 + h_4 - h_5$
$H(0) = \sqrt{2}$
$h_0 + h_1 + h_2 + h_3 + h_4 + h_5 = \sqrt{2}$

additional conditions on $H(\omega)$ will be

$H'(\pi) = 0$
& further flatten at $\omega = \pi$ by
$H'' = 0$

$\therefore\ H(\omega) = h_0 + h_1 e^{j\omega} + h_2 e^{2j\omega} + h_3 e^{3j\omega} + h_4 e^{4j\omega} + h_5 e^{5j\omega}$

$\therefore\ H'(\omega) = j \cdot h_1 e^{j\omega} + 2j \cdot h_2 e^{2j\omega} + 3j \cdot h_3 e^{3j\omega} + 4j \cdot h_4 e^{4j\omega} + 5j \cdot h_5 e^{5j\omega}$

using $j^2 = (\sqrt{-1})^2 = -1$

$$H''(\omega) = -h_1 e^{j\omega} - 4h_2 e^{2j\omega} - 9h_3 e^{3j\omega} - 16h_4 e^{4j\omega} - 25h_5 e^{5j\omega}$$

let's evaluate @ $\omega = \pi$
$\therefore H'(\pi) = 0 = h_1 - 2h_2 + 3h_3 - 4h_4 + 5h_5$

$\therefore H''(\pi) = 0 = h_1 - 4h_2 + 9h_3 - 16h_4 + 25h_5$

$\therefore$ complete set of design equations:

$$\begin{aligned} h_0^2 + h_1^2 + h_2^2 + h_3^2 + h_4^2 + h_5^2 &= 1 \\ h_0 h_2 + h_1 h_3 + h_2 h_4 + h_3 h_5 &= 0 \\ h_0 h_4 + h_1 h_5 &= 0 \\ h_0 - h_1 + h_2 - h_3 + h_4 - h_5 &\\ h_1 - 2h_2 + 3h_3 - 4h_4 + 5h_5 &\\ h_1 - 4h_2 + 9h_3 - 16h_4 + 25h_5 & \end{aligned} \tag{D4}$$

solving we get daub-6 coefficients

$$h_0 = \frac{\sqrt{2}}{32}\left(1 + \sqrt{10} + \sqrt{5 + 2\sqrt{10}}\right)$$
$$h_1 = \frac{\sqrt{2}}{32}\left(5 + \sqrt{10} + 3\sqrt{5 + 2\sqrt{10}}\right)$$
$$h_2 = \frac{\sqrt{2}}{32}\left(10 - 2\sqrt{10} + 2\sqrt{5 + 2\sqrt{10}}\right)$$
$$h_3 = \frac{\sqrt{2}}{32}\left(10 - \sqrt{10} - 2\sqrt{5 + 2\sqrt{10}}\right)$$
$$h_4 = \frac{\sqrt{2}}{32}\left(5 + \sqrt{10} - 3\sqrt{5 + 2\sqrt{10}}\right)$$
$$h_5 = \frac{\sqrt{2}}{32}\left(1 + \sqrt{10} + \sqrt{5 + 2\sqrt{10}}\right)$$

$$\&\quad g_k = (-1)^k h_5 - k,\ k = 0, \cdots 5$$

$$g_0 = h_5 = \frac{\sqrt{2}}{32}\left(1+\sqrt{10}+\sqrt{5+2\sqrt{10}}\right)$$

$$g_1 = -h_4 = -\frac{\sqrt{2}}{32}\left(5+\sqrt{10}-3\sqrt{5+2\sqrt{10}}\right)$$

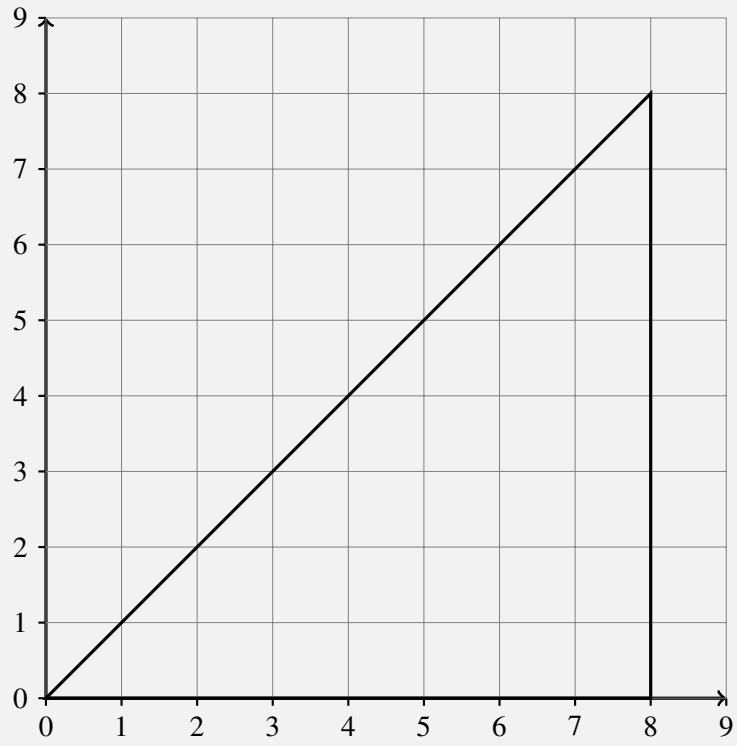
$$g_2 = h_3 = \frac{\sqrt{2}}{32}\left(10-\sqrt{10}-2\sqrt{5+2\sqrt{10}}\right)$$

$$g_3 = -h2 = -\frac{\sqrt{2}}{32}\left(10-2\sqrt{10}+2\sqrt{5+2\sqrt{10}}\right)$$

$$g_4 = h_1 = \frac{\sqrt{2}}{32}\left(5+\sqrt{10}+3\sqrt{5+2\sqrt{10}}\right)$$

$$g_5 = -h_0 = \frac{\sqrt{2}}{32}\left(1+\sqrt{10}+\sqrt{5+2\sqrt{10}}\right)$$

Analytics 4.13.1 — Linear Sarga - Vedānta Style. **Illustration:**:Use Haar wavelet transform on following signal and solve given objectives:



$$f(x) = \begin{cases} x, & 0 \geq x \geq 8 \\ 0, & \text{otherwise} \end{cases}$$

1. $f_0(x) \in v_0$
2. $g_0(x) \in w_0$
3. $f_1(x) \in v_1$
4. Prove: $v_1 = v_0 \oplus w_0$

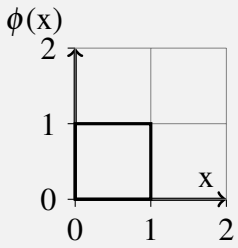
Directives:To solve this problem,we are going to use haar wavelet.The Scaling function of Haar wavelet is:

$f_j(x) \in v_j$, $w_a = \frac{1}{2^j}$

$span\left\{2^{\frac{j}{2}}\phi(2^j x-k)\right\}$

$$f_j(x) = \sum_k \alpha_{j,k} 2^{\frac{j}{2}} \phi(2^j x-k) \tag{4.55}$$

$$\alpha_{j,k} = \int_{-\infty}^{\infty} f_j(x) 2^{\frac{j}{2}} \phi(2^j x-k) \tag{4.56}$$

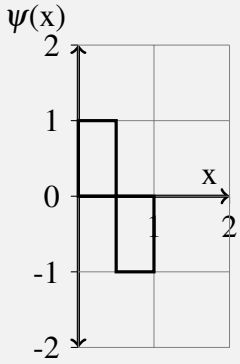


The dilation function of Haar wavelet is: $g_j(x) \in w_j$, $w_a = \frac{1}{2^j}$

$span\left\{2^{\frac{j}{2}}\psi(2^j x-k)\right\}$

$$g_j(x) = \sum_k \beta_{j,k} 2^{\frac{j}{2}} \psi(2^j x-k) \tag{4.57}$$

$$\beta_{j,k} = \int_{-\infty}^{\infty} g_j(x) 2^{\frac{j}{2}} \psi(2^j x-k) \tag{4.58}$$



1] $f_0(x) \in v_0 \therefore j = 0$ so $w_a = \frac{1}{2^0} = 1$
$span\left\{2^{\frac{0}{2}}\phi(2^0x-k)\right\}$
$span\{\phi(x-k)\}$

From (2),when we put j=0 we get

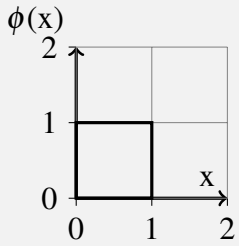
$$\begin{aligned} f_0(x) &= \sum_k \alpha_{0,k}(2)^{\frac{0}{2}}\phi(2^0x-k) \\ &= \sum_k \alpha_{0,k}\phi(x-k) \end{aligned} \tag{4.59}$$

$$\begin{aligned} \alpha_{0,k} &= \int_{-\infty}^{\infty} f_0(x)2^{\frac{0}{2}}\phi(2^0x-k) \\ &= \int_{-\infty}^{\infty} f_0(x)\phi(x-k) \end{aligned} \tag{4.60}$$

Putting k=0 in equation (6),

$$\begin{aligned} \alpha_{0,0} &= \int_{-\infty}^{\infty} f_0(x)\phi(x-0)dx \\ &= \int_{-\infty}^{\infty} f_0(x)\phi(x)dx \end{aligned} \tag{4.61}$$

But,

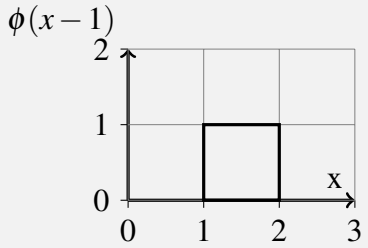


So scaling function $\phi(x)$ varies from 0 to 1.
$= \int_0^1 (x)(1)dx$
$= \left[\frac{x^2}{2}\right]_0^1$
$= \frac{1}{2}$
$\boxed{\therefore \alpha_{0,0} = \frac{1}{2}}$

Putting k=1 in equation (6),

$$\begin{aligned} \alpha_{0,1} &= \int_{-\infty}^{\infty} f_0(x)\phi(x-1)dx \\ &= \int_{-\infty}^{\infty} f_0(x)\phi(x-1)dx \end{aligned} \tag{4.62}$$

But,

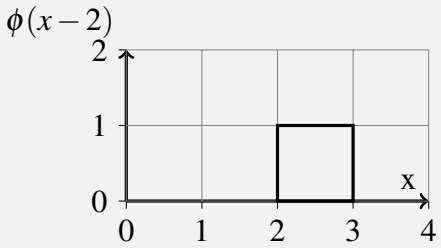


So scaling function $\phi(x-1)$ varies from 1 to 2.
$= \int_1^2 (x)(1)dx$
$= \left[\frac{x^2}{2}\right]_1^2$
$= \frac{3}{2}$
$\boxed{\therefore \alpha_{0,1} = \frac{3}{2}}$

Putting k=2 in equation (6),

$$\begin{aligned}\alpha_{0,2} &= \int_{-\infty}^{\infty} f_0(x)\phi(x-2)dx \\ &= \int_{-\infty}^{\infty} f_0(x)\phi(x-2)dx\end{aligned} \tag{4.63}$$

But,



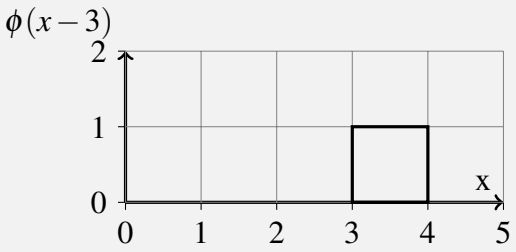
So scaling function $\phi(x-2)$ varies from 2 to 3.
$= \int_2^3 (x)(1)dx$
$= \left[\frac{x^2}{2}\right]_2^3$
$= \frac{5}{2}$

$$\boxed{\therefore \alpha_{0,2} = \frac{5}{2}}$$

For k=3,

$$\begin{aligned}\alpha_{0,3} &= \int_{-\infty}^{\infty} f_0(x)\phi(x-3)dx \\ &= \int_{-\infty}^{\infty} f_0(x)\phi(x-3)dx\end{aligned} \tag{4.64}$$

But,

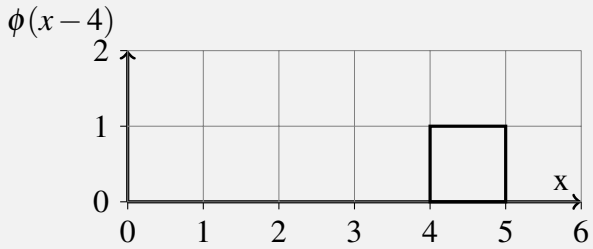


So scaling function $\phi(x-3)$ varies from 3 to 4.
$= \int_3^4 (x)(1)dx$
$= \left[\frac{x^2}{2}\right]_3^4$
$= \frac{7}{2}$
$\boxed{\therefore \alpha_{0,3} = \frac{7}{2}}$

For k=4,

$$\begin{aligned} \alpha_{0,4} &= \int_{-\infty}^{\infty} f_0(x)\phi(x-4)dx \\ &= \int_{-\infty}^{\infty} f_0(x)\phi(x-4)dx \end{aligned} \tag{4.65}$$

But,

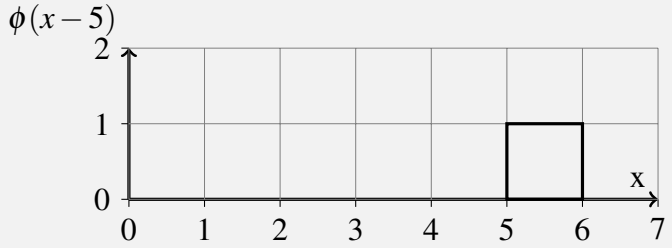


So scaling function $\phi(x-4)$ varies from 4 to 5.
$= \int_4^5 (x)(1)dx$
$= \left[\frac{x^2}{2}\right]_4^5$
$= \frac{9}{2}$
$\boxed{\therefore \alpha_{0,4} = \frac{9}{2}}$

For k=5,

$$\begin{aligned} \alpha_{0,5} &= \int_{-\infty}^{\infty} f_0(x)\phi(x-5)dx \\ &= \int_{-\infty}^{\infty} f_0(x)\phi(x-5)dx \end{aligned} \tag{4.66}$$

But,

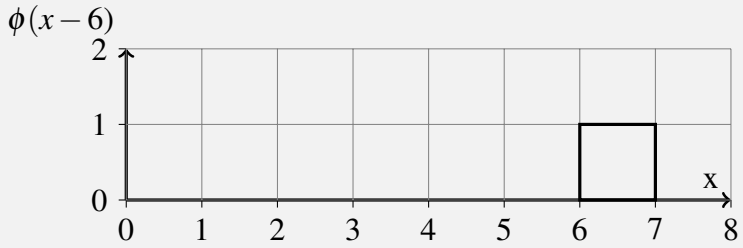


So scaling function $\phi(x-5)$ varies from 5 to 6.
$= \int_5^6 (x-4)(1)dx$
$= \left[\frac{x^2}{2} - 4 \cdot x\right]_5^6$
$= \frac{11}{2}$
$\boxed{\therefore \alpha_{0,5} = \frac{11}{2}}$

For k=6,

$$\begin{aligned} \alpha_{0,6} &= \int_{-\infty}^{\infty} f_0(x)\phi(x-6)dx \\ &= \int_{-\infty}^{\infty} f_0(x)\phi(x-6)dx \end{aligned} \tag{4.67}$$

But,

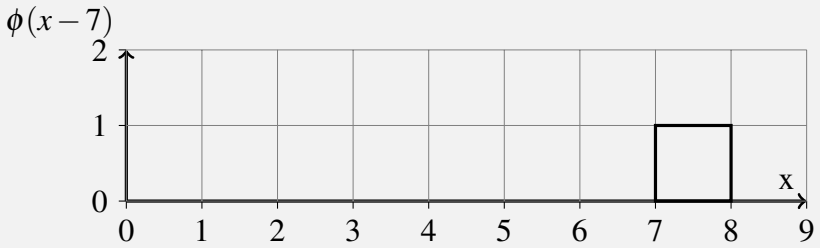


So scaling function $\phi(x-6)$ varies from 6 to 7.
$= \int_6^7 (x)(1)dx$
$= \left[\frac{x^2}{2}\right]_6^7$
$= \frac{13}{2}$
$\boxed{\therefore \alpha_{0,6} = \frac{13}{2}}$

For k=7,

$$\begin{aligned}\alpha_{0,7} &= \int_{-\infty}^{\infty} f_0(x)\phi(x-7)dx \\ &= \int_{-\infty}^{\infty} f_0(x)\phi(x-7)dx\end{aligned} \tag{4.68}$$

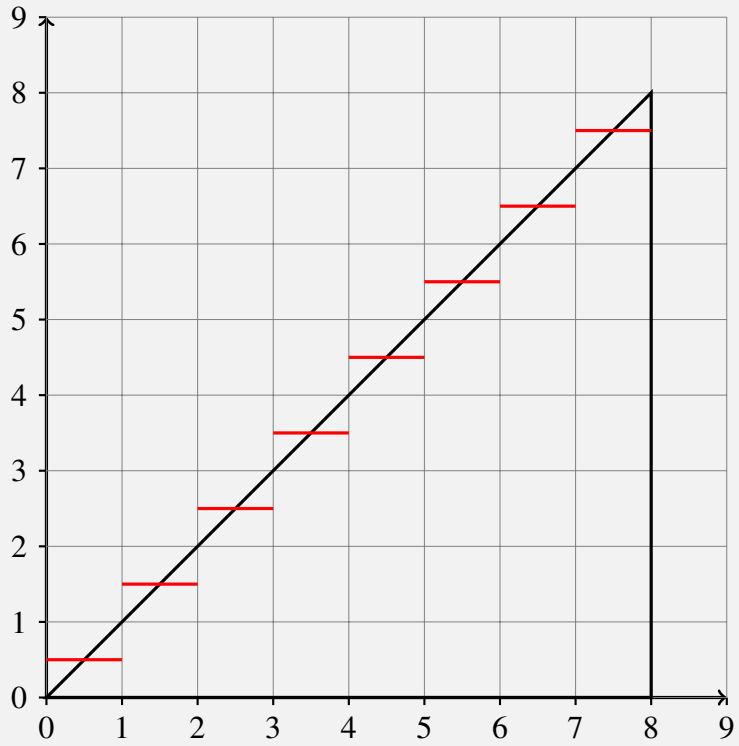
But,



So scaling function $\phi(x-7)$ varies from 7 to 8.
$= \int_7^8 (x)(1)dx$
$= \left[\frac{x^2}{2}\right]_7^8$
$= \frac{15}{2}$

$$\boxed{\therefore \alpha_{0,7} = \frac{15}{2}}$$

Using equation (5),we can write
$f_0(x) = \frac{1}{2}\phi(x) + \frac{3}{2}\phi(x-1) + \frac{5}{2}\phi(x-2) + \frac{7}{2}\phi(x-3) + \frac{9}{2}\phi(x-4) + \frac{11}{2}\phi(x-5) + \frac{13}{2}\phi(x-6) + \frac{15}{2}\phi(x-7)$



The red line is showing Wavelet scaling coefficients.

2]$g_0(x) \in g_0 \therefore j = 0$ so $w_a = \frac{1}{2^0} = 1$

$$span\left\{2^{\frac{0}{2}}\psi(2^0x-k)\right\} \\ span\{\psi(x-k)\} \tag{4.69}$$

From (3) & (4),when we put j=0

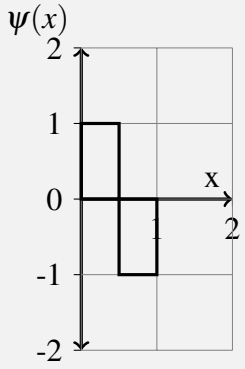
$$f_0(x) = \sum_k \beta_{0,k}(2)^{\frac{0}{2}}\psi(2^0x-k) \\ = \sum_k \beta_{0,k}\psi(x-k) \tag{4.70}$$

$$\beta_{0,k} = \int_{-\infty}^{\infty} f_0(x)2^{\frac{0}{2}}\psi(2^0x-k) \\ = \int_{-\infty}^{\infty} f_0(x)\psi(x-k) \tag{4.71}$$

For k=0,

$$\begin{aligned}\beta_{0,0} &= \int_{-\infty}^{\infty} f_0(x)\psi(x-0)dx \\ &= \int_{-\infty}^{\infty} f_0(x)\psi(x)dx\end{aligned} \tag{4.72}$$

But,



So dilation function $\psi(x)$ varies from 0 to 1.

$= \int_0^{\frac{1}{2}}(x)(1) + \int_{\frac{1}{2}}^{1}(x)(-1)dx$

$= \left[fracx^2 2\right]_0^{\frac{1}{2}} + \left[-\left(-\frac{x^2}{2}\right)\right]_{\frac{1}{2}}^{1}$

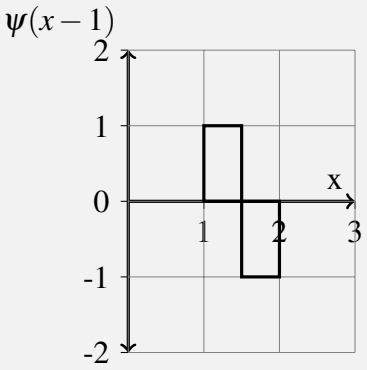
$= -\frac{1}{4}$

$$\boxed{\therefore \beta_{0,0} = -\frac{1}{4}}$$

For k=1,

$$\begin{aligned}\beta_{0,1} &= \int_{-\infty}^{\infty} f_0(x)\psi(x-1)dx \\ &= \int_{-\infty}^{\infty} f_0(x)\psi(x-1)dx\end{aligned} \tag{4.73}$$

But,



So dilation function $\psi(x-1)$ varies from 1 to 2.

$= \int_0^{\frac{1}{2}}(x)(1) + \int_{\frac{1}{2}}^{1}(x)(-1)dx$

$= \left[\frac{x^2}{2}\right]_1^{\frac{3}{2}} + \left[-\left(-\frac{x^2}{2}\right)\right]_{\frac{3}{2}}^{2}$

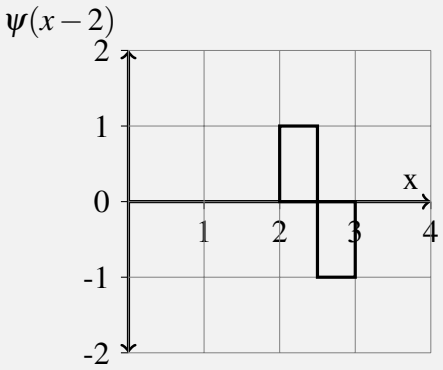
$= -\frac{1}{4}$

$$\therefore \beta_{0,1} = -\frac{1}{4}$$

For k=2,

$\beta_{0,2} = \int_{-\infty}^{\infty} f_0(x)\psi(x-2)dx$

But,



So dilation function $\psi(x-2)$ varies from 2 to 3.

$= \int_2^{\frac{5}{2}}(x)(1) + \int_{\frac{5}{2}}^{3}(x)(-1)dx$

$= \left[\frac{x^2}{2}\right]_2^{\frac{5}{2}} + \left[-\left(\frac{x^2}{2}\right)\right]_{\frac{5}{2}}^{3}$

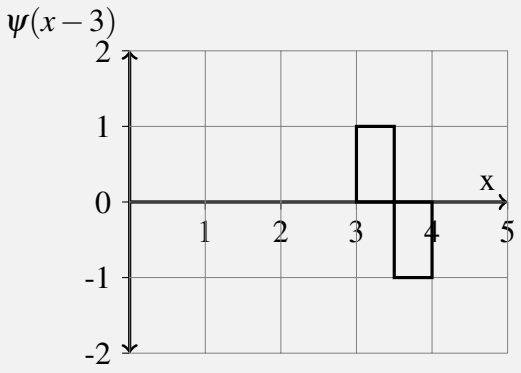
$= -\frac{1}{4}$

$$\therefore \beta_{0,2} = -\frac{1}{4}$$

For k=3,

$$\beta_{0,3} = \int_{-\infty}^{\infty} f_0(x)\psi(x-3)dx$$

But,



So dilation function $\psi(x-3)$ varies from 3 to 4.

$= \int_3^{\frac{7}{2}}(x)(1) + \int_{\frac{7}{2}}^{4}(x)(-1)dx$

$= \left[\frac{x^2}{2}\right]_3^{\frac{7}{2}} + \left[-\left(\frac{x^2}{2}\right)\right]_{\frac{7}{2}}^{4}$

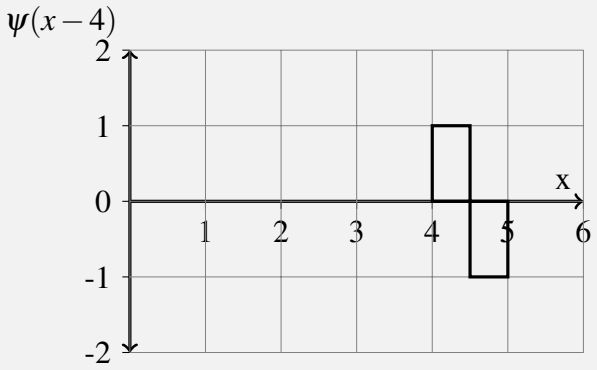
$= -\frac{1}{4}$

$$\therefore \beta_{0,3} = -\frac{1}{4}$$

For k=4,

$$\beta_{0,4} = \int_{-\infty}^{\infty} f_0(x)\psi(x-4)dx$$

But,



So dilation function $\psi(x-4)$ varies from 4 to 5.

$= \int_4^{\frac{9}{2}}(x)(1) + \int_{\frac{9}{2}}^{5}(x)(-1)dx$

$= \left[\frac{x^2}{2}\right]_4^{\frac{9}{2}} + \left[-\left(\frac{x^2}{2}\right)\right]_{\frac{9}{2}}^{5}$

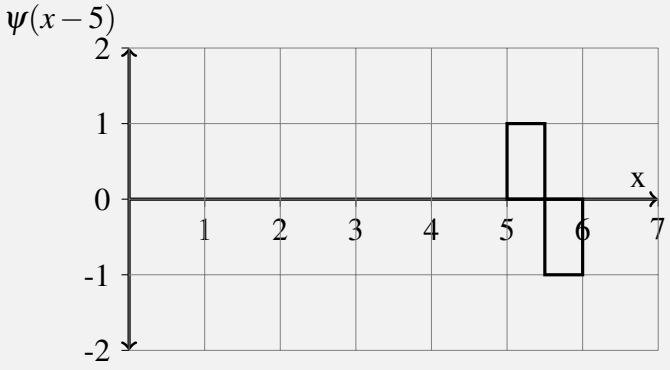
$= -\frac{1}{4}$

$$\therefore \beta_{0,4} = -\frac{1}{4}$$

For k=5,

$$\beta_{0,5} = \int_{-\infty}^{\infty} f_0(x)\psi(x-5)dx$$

But,



So dilation function $\psi(x-5)$ varies from 5 to 6.

$= \int_5^{\frac{11}{2}}(x)(1) + \int_{\frac{11}{2}}^{6}(x)(-1)dx$

$= \left[\frac{x^2}{2}-\right]_5^{\frac{11}{2}} + \left[-\left(\frac{x^2}{2}-\right)\right]_{\frac{11}{2}}^{6}$

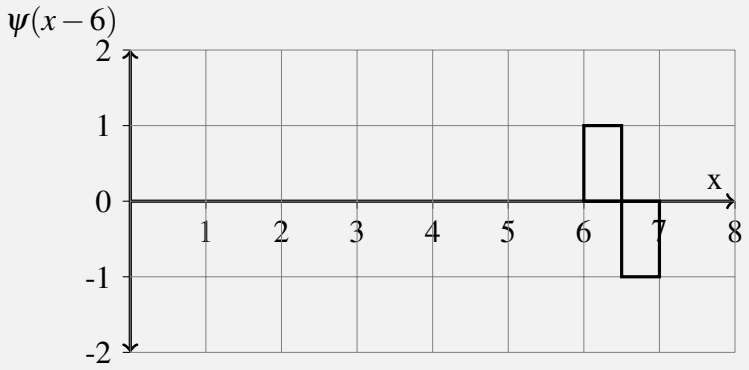
$= -\frac{1}{4}$

$$\therefore \beta_{0,5} = -\frac{1}{4}$$

For k=6,

$\beta_{0,6} = \int_{-\infty}^{\infty} f_0(x)\psi(x-6)dx$

But,



So dilation function $\psi(x-6)$ varies from 6 to 7.

$= \int_6^{\frac{13}{2}} (x)(1) + \int_{\frac{13}{2}}^{7} (x)(-1)dx$

$= \left[\frac{x^2}{2}\right]_6^{\frac{13}{2}} + \left[-\left(\frac{x^2}{2}\right)\right]_{\frac{13}{2}}^{7}$

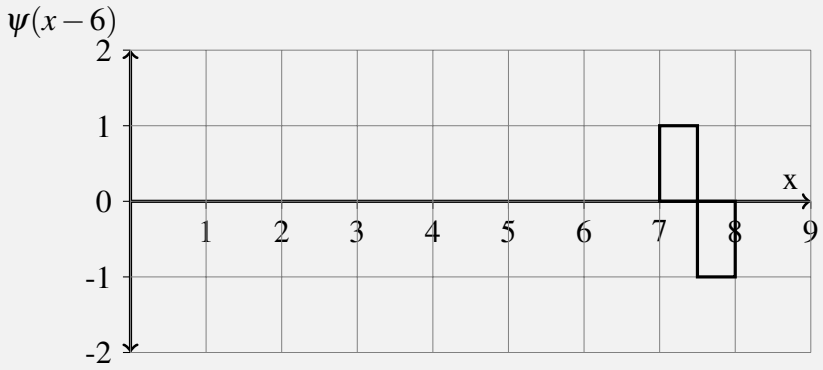
$= -\frac{1}{4}$

$$\therefore \beta_{0,6} = -\frac{1}{4}$$

For k=7,

$\beta 0,7 = \int_{-\infty}^{\infty} f_0(x)\psi(x-7)dx$

But,



So dilation function $\psi(x-7)$ varies from 7 to 8.

$= \int_7^{\frac{15}{2}} (x)(1) + \int_{\frac{14}{2}}^{8} (x)(-1)dx$

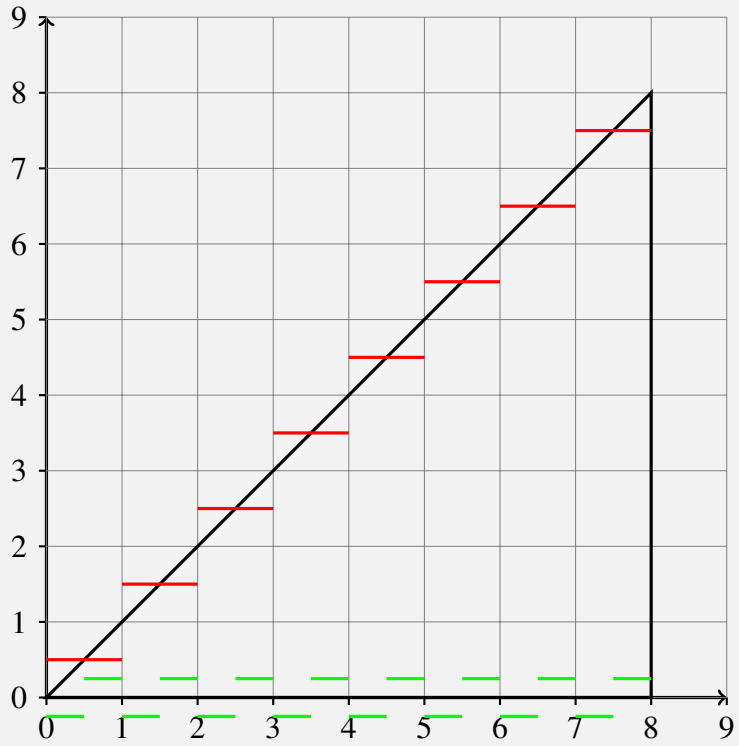
$= \left[\frac{x^2}{2}\right]_7^{\frac{15}{2}} + \left[-\left(\frac{x^2}{2}\right)\right]_{\frac{15}{2}}^{8}$

$= -\frac{1}{4}$

$$\therefore \beta_{0,7} = -\frac{1}{4}$$

Using high pass projection equation,we can write

$g_0(x) = -\frac{1}{4}\psi(x) - \frac{1}{4}\psi(x-1) - \frac{1}{4}\psi(x-2) - \frac{1}{4}\phi(x-3) - \frac{1}{4}\phi(x-4) - \frac{1}{4}\phi(x-5) - \frac{1}{4}\phi(x-6) - \frac{1}{4}\phi(x-7)$



The Red line is showing Wavelet scaling coefficients.
The Green line is showing Wavelet dilation coefficients.

3] $f_1(x) \in v_1 \therefore j = 1$ so $w_a = \frac{1}{2^1} = \frac{1}{2}$

$span\left\{2^{\frac{1}{2}}\phi(2^0x-k)\right\}$

$span\left\{\sqrt{2}\phi(2x-k)\right\}$

From (2),when we put j=1

$f_1(x) = \sum_k \alpha_{1,k}(2)^{\frac{1}{2}}\phi(2^1x-k)$

$= \sum_k \alpha_{1,k}\sqrt{2}\phi(2x-k)$

$\alpha_{1,k} = \int_{-\infty}^{\infty} f_1(x)2^{\frac{0}{2}}\phi(2^0x-k)$

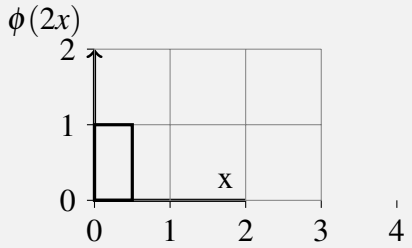
$$\alpha_{1,k} = \int_{-\infty}^{\infty} f_1(x)\phi(2x-k) \tag{4.74}$$

For k=0,

$\alpha_{1,0} = \int_{-\infty}^{\infty} f_1(x)\sqrt{2}\phi(2x-0)dx$

$= \int_{-\infty}^{\infty} f_1(x)\sqrt{2}\phi(2x)dx$

But,



As scaling function $\phi(2x)$ varies from 0 to 0.5.

$= \int_0^{\frac{1}{2}} \sqrt{2}(x)(1)dx$

$= \sqrt{2}\left[\frac{x^2}{2}\right]_{\frac{1}{2}}^{0}$

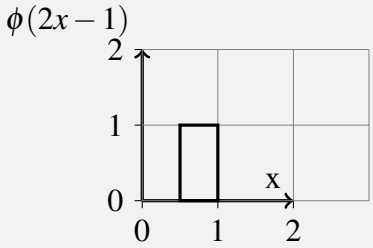
$= \sqrt{2} \cdot \frac{1}{8}$

$$\boxed{\therefore \alpha_{1,0} = \frac{1}{4\sqrt{2}}}$$

For k=1,

$\alpha_{1,1} = \int_{-\infty}^{\infty} \sqrt{2} f_1(x)\phi(2x-1)dx$

But,



As scaling function $\phi(2x)$ varies from 0.5 to 1.

$= \int_0^{\frac{0.5}{1}} \sqrt{2}(x)(1)dx$

$= \sqrt{2}\left[\frac{x^2}{2}\right]_{\frac{0.5}{1}}^{0}$

$= \sqrt{2} \cdot \frac{3}{8}$

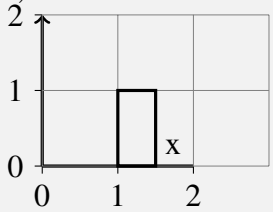
$$\therefore \alpha_{1,1} = \frac{3}{4\sqrt{2}}$$

For k=2,

$$\alpha_{1,2} = \int_{-\infty}^{\infty} f_1(x)\sqrt{2}\phi(2x-2)dx$$

But,

$\phi(2x-2)$



As scaling function $\phi(2x-2)$ varies from 1 to $\frac{3}{2}$.

$= \int_1^{\frac{3}{2}} \sqrt{2}(x)(1)dx$

$= \sqrt{2}\left[\frac{x^2}{2}\right]_{\frac{3}{2}}^{1}$

$= \sqrt{2} \cdot \frac{5}{8}$

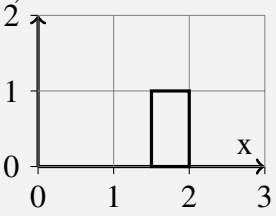
$$\therefore \alpha_{1,2} = \frac{5}{4\sqrt{2}}$$

For k=3,

$$\alpha_{1,3} = \int_{-\infty}^{\infty} f_1(x)\sqrt{2}\phi(2x-3)dx$$

But,

$\phi(2x-3)$



As scaling function $\phi(2x-3)$ varies from 1.5 to 2.

$= \int_{\frac{3}{2}}^{2} \sqrt{2}(4-x)(1)dx$

$= \sqrt{2}\left[\frac{x^2}{2}\right]_2^{\frac{3}{2}}$

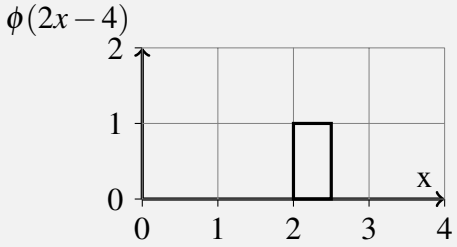
$= \sqrt{2} \cdot \frac{7}{8}$

$$\boxed{\therefore \alpha_{1,3} = \frac{7}{4\sqrt{2}}}$$

For k=4,

$$\alpha_{1,4} = \int_{-\infty}^{\infty} f_1(x)\sqrt{2}\phi(2x-4)dx$$

But,



As scaling function $\phi(2x-4)$ varies from 2 to $\frac{5}{2}$.

$= \int_{2}^{\frac{5}{2}} \sqrt{2}(x)(1)dx$

$= \sqrt{2}\left[\frac{x^2}{2}\right]_{\frac{5}{2}}^{2}$

$= \sqrt{2} \cdot \frac{9}{8}$

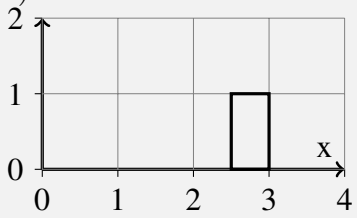
$$\boxed{\therefore \alpha_{1,4} = \frac{9}{4\sqrt{2}}}$$

For k=5,

$$\alpha_{1,5} = \int_{-\infty}^{\infty} f_1(x)\sqrt{2}\phi(2x-5)dx$$

But,

$\phi(2x-5)$



As scaling function $\phi(2x-5)$ varies from $\frac{5}{2}$ to 3.

$= \int_{\frac{5}{2}}^{6} \sqrt{2}(x)(1)dx$

$= \sqrt{2}\left[\frac{x^2}{2}\right]_{\frac{5}{2}}^{3}$

$= \sqrt{2} \cdot \frac{11}{8}$

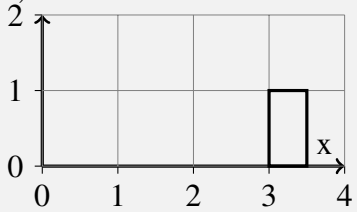
$$\therefore \alpha_{1,5} = \frac{11}{4\sqrt{2}}$$

For k=6,

$$\alpha_{1,6} = \int_{-\infty}^{\infty} f_1(x)\sqrt{2}\phi(2x-6)dx$$

But,

$\phi(2x-6)$



As scaling function $\phi(2x-6)$ varies from 3 to $\frac{7}{2}$.

$= \int_{3}^{\frac{7}{2}} \sqrt{2}(x)(1)dx$

$= \sqrt{2}\left[\frac{x^2}{2}\right]_{\frac{7}{2}}^{3}$

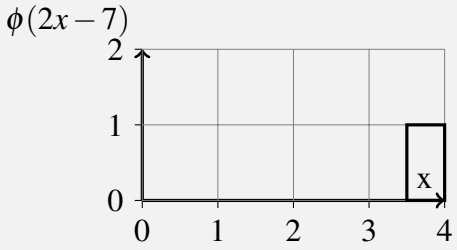
$= \sqrt{2} \cdot \frac{13}{8}$

$$\therefore \alpha_{1,4} = \frac{13}{4\sqrt{2}}$$

For k=7,

$$\alpha_{1,7} = \int_{-\infty}^{\infty} f_1(x)\sqrt{2}\phi(2x-7)dx$$

But,

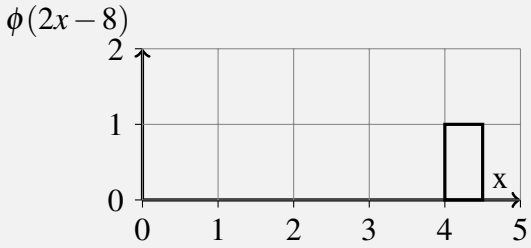


As scaling function $\phi(2x-7)$ varies from $\frac{7}{2}$ to 4.
$= \int_{\frac{7}{2}}^{4} \sqrt{2}(x)(1)dx$
$= \sqrt{2}\left[\frac{x^2}{2}\right]_{\frac{7}{2}}^{4}$
$= \sqrt{2} \cdot \frac{15}{8}$
$\boxed{\therefore \alpha_{1,7} = \frac{15}{4\sqrt{2}}}$

For k=8,

$\alpha_{1,8} = \int_{-\infty}^{\infty} f_1(x)\sqrt{2}\phi(2x-8)dx$

But,



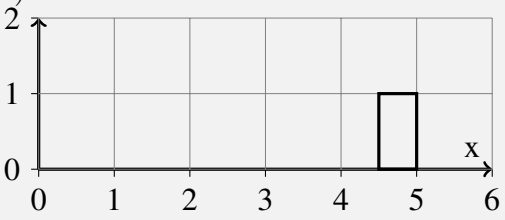
As scaling function $\phi(2x-8)$ varies from 4 to $\frac{9}{2}$.
$= \int_{4}^{\frac{9}{2}} \sqrt{2}(x)(1)dx$
$= \sqrt{2}\left[\frac{x^2}{2}\right]_{\frac{9}{2}}^{4}$
$= \sqrt{2} \cdot \frac{17}{8}$
$\boxed{\therefore \alpha_{1,8} = \frac{17}{4\sqrt{2}}}$

For k=9,

$$\alpha_{1,9} = \int_{-\infty}^{\infty} f_1(x)\sqrt{2}\phi(2x-9)dx$$

But,

$\phi(2x-9)$



As scaling function $\phi(2x-9)$ varies from $\frac{9}{2}$ to 5.

$= \int_{\frac{9}{2}}^{5} \sqrt{2}(x)(1)dx$

$= \sqrt{2}\left[\frac{x^2}{2}\right]_{\frac{9}{2}}^{5}$

$= \sqrt{2} \cdot \frac{19}{8}$

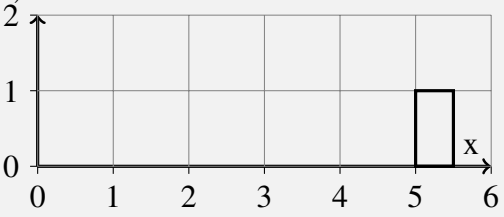
$$\therefore \alpha_{1,9} = \frac{19}{4\sqrt{2}}$$

For k=10,

$$\alpha_{1,10} = \int_{-\infty}^{\infty} f_1(x)\sqrt{2}\phi(2x-10)dx$$

But,

$\phi(2x-10)$



As scaling function $\phi(2x-10)$ varies from 5 to $\frac{11}{2}$.

$= \int_{5}^{\frac{11}{2}} \sqrt{2}(x)(1)dx$

$= \sqrt{2}\left[\frac{x^2}{2}\right]_{\frac{11}{2}}^{5}$

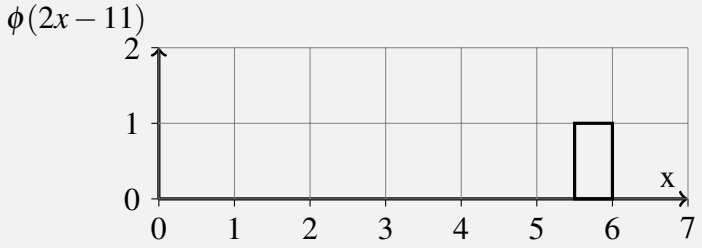
$= \sqrt{2} \cdot \frac{21}{8}$

$$\boxed{\therefore \alpha_{1,10} = \frac{21}{4\sqrt{2}}}$$

For k=11,

$\alpha_{1,11} = \int_{-\infty}^{\infty} f_1(x)\sqrt{2}\phi(2x-11)dx$

But,



As scaling function $\phi(2x-11)$ varies from $\frac{11}{2}$ to 6.

$= \int_{\frac{11}{2}}^{6} \sqrt{2}(x)(1)dx$

$= \sqrt{2}\left[\frac{x^2}{2}\right]_{\frac{11}{2}}^{6}$

$= \sqrt{2} \cdot \frac{23}{8}$

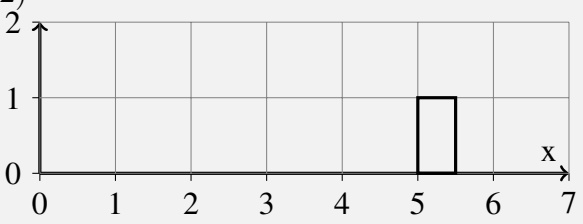
$$\boxed{\therefore \alpha_{1,11} = \frac{23}{4\sqrt{2}}}$$

For k=12,

$\alpha_{1,12} = \int_{-\infty}^{\infty} f_1(x)\sqrt{2}\phi(2x-12)dx$

But,

$\phi(2x-12)$



As scaling function $\phi(2x-10)$ varies from 5 to $\frac{11}{2}$.

$= \int_5^{\frac{11}{2}} \sqrt{2}(x)(1)dx$

$= \sqrt{2}\left[\frac{x^2}{2}\right]_{\frac{11}{2}}^{5}$

$= \sqrt{2} \cdot \frac{25}{8}$

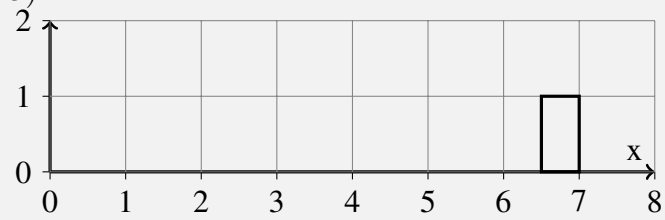
$$\therefore \alpha_{1,10} = \frac{25}{4\sqrt{2}}$$

For k=13,

$$\alpha_{1,13} = \int_{-\infty}^{\infty} f_1(x)\sqrt{2}\phi(2x-13)dx$$

But,

$\phi(2x-13)$



As scaling function $\phi(2x-13)$ varies from $\frac{13}{2}$ to 7.

$= \int_{\frac{13}{2}}^{7} \sqrt{2}(x)(1)dx$

$= \sqrt{2}\left[\frac{x^2}{2}\right]_{\frac{13}{2}}^{7}$

$= \sqrt{2} \cdot \frac{27}{8}$

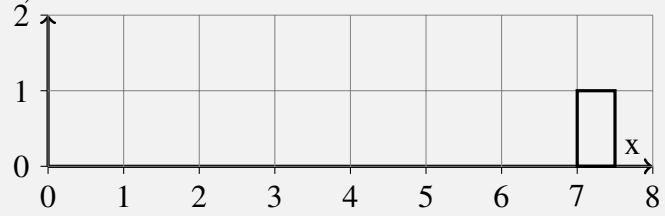
$$\therefore \alpha_{1,13} = \frac{27}{4\sqrt{2}}$$

For k=14,

$\alpha_{1,14} = \int_{-\infty}^{\infty} f_1(x)\sqrt{2}\phi(2x-14)dx$

But,

$\phi(2x-14)$



As scaling function $\phi(2x-14)$ varies from 7 to $\frac{15}{2}$.

$= \int_7^{\frac{15}{2}} \sqrt{2}(x)(1)dx$

$= \sqrt{2}\left[\frac{x^2}{2}\right]_{\frac{15}{2}}^{7}$

$= \sqrt{2}\cdot\frac{29}{8}$

$\boxed{\therefore \alpha_{1,14} = \frac{29}{4\sqrt{2}}}$

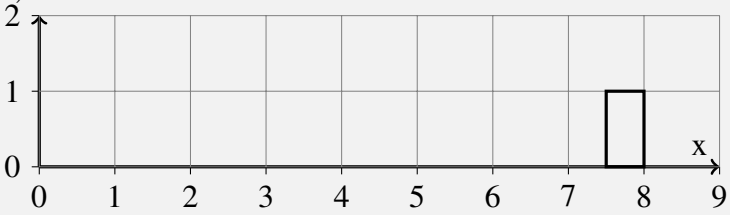
For k=15,

$\alpha_{1,15} = \int_{-\infty}^{\infty} f_1(x)\sqrt{2}\phi(2x-15)dx$

$= \int_{-\infty}^{\infty} f_1(x)\sqrt{2}\phi(2x-15)dx$

But,

$\phi(2x-15)$



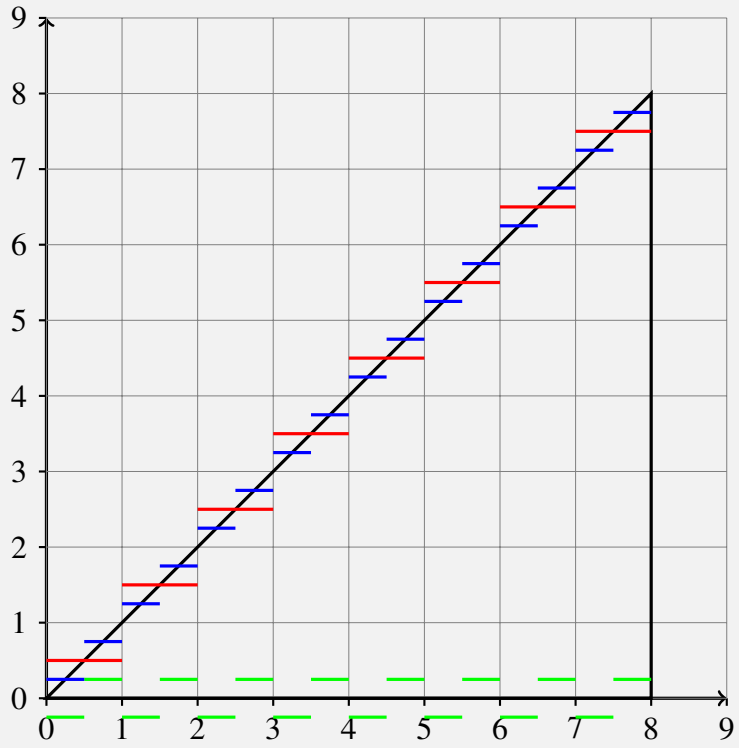
As scaling function $\phi(2x-15)$ varies from $\frac{15}{2}$ to 8.

$= \int_{\frac{15}{2}}^{8} \sqrt{2}(x)(1)dx$

$= \sqrt{2}\left[\frac{x^2}{2}\right]_{\frac{15}{2}}^{8}$

$= \sqrt{2}\cdot\frac{31}{8}$

$$\therefore \alpha_{1,15} = \frac{31}{4\sqrt{2}}$$



The Red line is showing Wavelet scaling coefficients.
The Green line is showing Wavelet dilation coefficients.

4] Prove: $v_1 = v_0 \oplus w_0$

$v_0 = \{\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}, \frac{9}{2}, \frac{11}{2}, \frac{13}{2}, \frac{15}{2}\}$

$w_0 = \{-\frac{1}{4}, -\frac{1}{4}, -\frac{1}{4}, -\frac{1}{4}, -\frac{1}{4}, -\frac{1}{4}, -\frac{1}{4}, -\frac{1}{4}\}$

$v_1 = \{\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{7}{4}, \frac{9}{4}, \frac{11}{4}, \frac{13}{4}, \frac{15}{4}, \frac{17}{4}, \frac{19}{4}, \frac{21}{4}, \frac{23}{4}, \frac{25}{4}, \frac{27}{4}, \frac{29}{4}, \frac{31}{4}, \}$

Now, Take first component of v_0 and first component of w_0.
and Find out following term,
$\alpha_{0,1} \cdot \phi(x) + \beta_{0,1} \cdot \psi(k)$
The Scaling function varies from 0 to 1 and the value is 1 over all interval. The dilation function varies from 0 to 1 and the value is 1 for 0 to 0.5 and -1 for 0.5 to 1. So, We will get two values using these two functions.
For K=0
$\therefore {\alpha_{0,0}}^{(+)} + {\beta_{0,0}}^{(+)}$
$= \frac{1}{2} + \frac{-1}{4}$

$=\frac{1}{4}$

This is equal to $\alpha_{1,0}$.

Also,
$\therefore {\alpha_{0,0}}^{(-)} + {\beta_{0,0}}^{(-)}$
$=\frac{1}{2}+\frac{1}{4}$
$=\frac{3}{4}$

This is equal to $\alpha_{1,1}$.

For K=1
$\therefore {\alpha_{0,1}}^{(+)} + {\beta_{0,1}}^{(+)}$
$=\frac{3}{2}+\frac{-1}{4}$
$=\frac{5}{4}$

This is equal to $\alpha_{1,2}$.

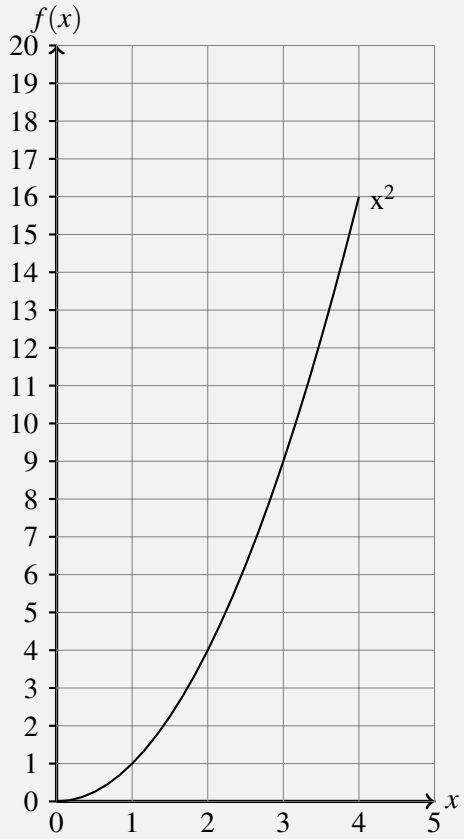
$\therefore {\alpha_{0,1}}^{(-)} + {\beta_{0,1}}^{(-)}$
$=\frac{3}{2}+\frac{-1}{4}$
$=\frac{7}{4}$

This is equal to $\alpha_{1,3}$.

Similarly, we can find values of $\alpha_{0,k}$ and $\beta_{0,k}$ to make a conclusion that addition of these two values will $\alpha_{1,k}$.(k=Total number of coefficients).
Thus, v_1 is formed by combining v_0 and w_0.

Hence,$v_1 = v_0 \oplus w_0$.

Analytics 4.13.2 — Squared (Non-linear) Sarga - Vedānta Style. Illustration:Use Haar wavelet transform on following signal and solve given objectives:



$$f(x) = \begin{cases} x^2, & 0 \geq x \geq 4 \\ 0, & \text{otherwise} \end{cases}$$

1. $f_0(x) \in v_0$
2. $g_0(x) \in w_0$
3. $f_1(x) \in v_1$
4. Prove: $v_1 = v_0 \oplus w_0$

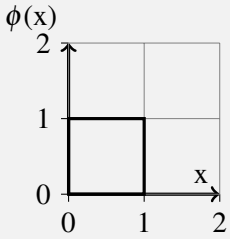
Directive:To solve this problem,we are going to use haar wavelet.The Scaling function of Haar wavelet is:

$$f_j(x) \in v_j,\ w_a = \frac{1}{2^j}$$

$$span\left\{2^{\frac{j}{2}}\phi(2^j x - k)\right\}$$

$$f_j(x) = \sum_k \alpha_{j,k} 2^{\frac{j}{2}} \phi(2^j x - k) \tag{4.75}$$

$$\alpha_{j,k} = \int_{-\infty}^{\infty} f_j(x) 2^{\frac{j}{2}} \phi(2^j x - k) \tag{4.76}$$

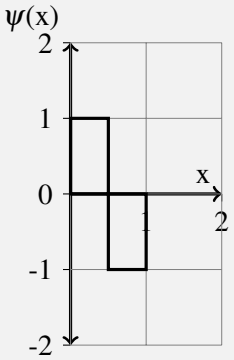


The dilation function of Haar wavelet is: $g_j(x) \in w_j$, $w_a = \frac{1}{2^j}$

$span\left\{2^{\frac{j}{2}} \psi(2^j x - k)\right\}$

$$g_j(x) = \sum_k \beta_{j,k} 2^{\frac{j}{2}} \psi(2^j x - k) \tag{4.77}$$

$$\beta_{j,k} = \int_{-\infty}^{\infty} g_j(x) 2^{\frac{j}{2}} \psi(2^j x - k) \tag{4.78}$$



1] $f_0(x) \in v_0 \therefore j = 0$ so $w_a = \frac{1}{2^0} = 1$

$span\left\{2^{\frac{0}{2}} \phi(2^0 x - k)\right\}$

$span\{\phi(x - k)\}$

From (2),when we put j=0 we get

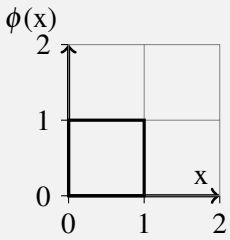
$$\begin{aligned} f_0(x) &= \sum_k \alpha_{0,k}(2)^{\frac{0}{2}}\phi(2^0x-k) \\ &= \sum_k \alpha_{0,k}\phi(x-k) \end{aligned} \tag{4.79}$$

$$\begin{aligned} \alpha_{0,k} &= \int_{-\infty}^{\infty} f_0(x)2^{\frac{0}{2}}\phi(2^0x-k) \\ &= \int_{-\infty}^{\infty} f_0(x)\phi(x-k) \end{aligned} \tag{4.80}$$

Putting k=0 in equation (6),

$$\begin{aligned} \alpha_{0,0} &= \int_{-\infty}^{\infty} f_0(x)\phi(x-0)dx \\ &= \int_{-\infty}^{\infty} f_0(x)\phi(x)dx \end{aligned} \tag{4.81}$$

But,

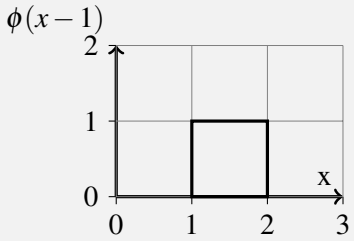


So scaling function $\phi(x)$ varies from 0 to 1.
$= \int_0^1 (x^2)(1)dx$
$= \left[\frac{x^3}{3}\right]_0^1$
$= \frac{1}{3}$
$\boxed{\therefore \alpha_{0,0} = \frac{1}{3}}$

Putting k=1 in equation (6),

$$\begin{aligned} \alpha_{0,1} &= \int_{-\infty}^{\infty} f_0(x)\phi(x-1)dx \\ &= \int_{-\infty}^{\infty} f_0(x)\phi(x-1)dx \end{aligned} \tag{4.82}$$

But,

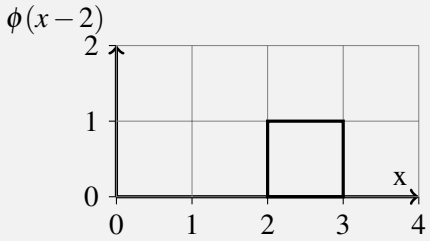


So scaling function $\phi(x-1)$ varies from 1 to 2.
$= \int_1^2 (x^2)(1)dx$
$= \left[\frac{x^3}{3}\right]_1^2$
$= \frac{7}{3}$
$\boxed{\therefore \alpha_{0,1} = \frac{7}{3}}$

Putting k=2 in equation (6),

$$\begin{aligned} \alpha_{0,2} &= \int_{-\infty}^{\infty} f_0(x)\phi(x-2)dx \\ &= \int_{-\infty}^{\infty} f_0(x)\phi(x-2)dx \end{aligned} \tag{4.83}$$

But,

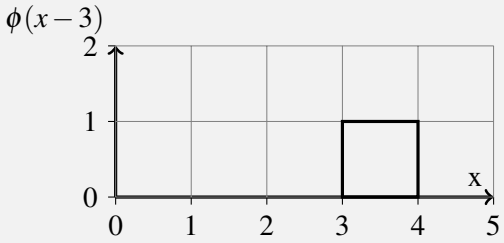


So scaling function $\phi(x-2)$ varies from 2 to 3.
$= \int_2^3 (x^2)(1)dx$
$= \left[\frac{x^3}{3}\right]_2^3$
$= \frac{19}{3}$
$\boxed{\therefore \alpha_{0,2} = \frac{19}{3}}$

For k=3,

$$\begin{aligned}\alpha_{0,3} &= \int_{-\infty}^{\infty} f_0(x)\phi(x-3)dx \\ &= \int_{-\infty}^{\infty} f_0(x)\phi(x-3)dx\end{aligned} \tag{4.84}$$

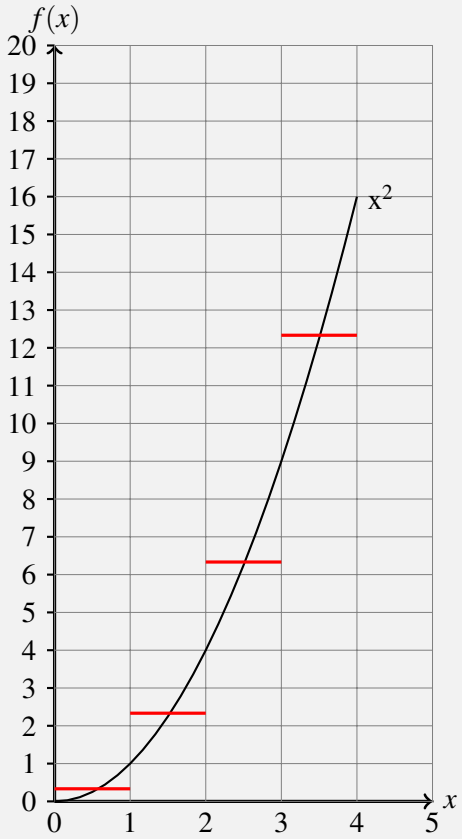
But,



So scaling function $\phi(x-3)$ varies from 3 to 4.
$= \int_3^4 (x^2)(1)dx$
$= \left[\frac{x^3}{3}\right]_3^4$
$= \frac{37}{3}$

$$\therefore \alpha_{0,3} = \frac{37}{3}$$

Using equation (5),we can write
$f_0(x) = \frac{1}{3}\cdot\phi(x) + \frac{7}{3}\cdot\phi(x-1) + \frac{19}{3}\cdot\phi(x-2) + \frac{37}{3}\cdot\phi(x-3)$



The red line is showing Wavelet scaling coefficients.
2]$g_0(x) \in g_0 \therefore j = 0$ so $w_a = \frac{1}{2^0} = 1$

$$span\left\{2^{\frac{0}{2}}\psi(2^0x-k)\right\} \\ span\{\psi(x-k)\} \tag{4.85}$$

From (3) & (4),when we put j=0

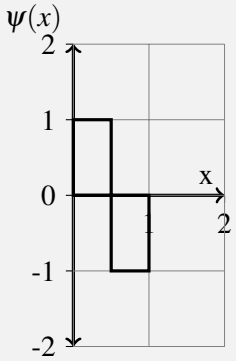
$$f_0(x) = \sum_k \beta_{0,k}(2)^{\frac{0}{2}}\psi(2^0x-k) \\ = \sum_k \beta_{0,k}\psi(x-k) \tag{4.86}$$

$$\beta_{0,k} = \int_{-\infty}^{\infty} f_0(x)2^{\frac{0}{2}}\psi(2^0x-k) \\ = \int_{-\infty}^{\infty} f_0(x)\psi(x-k) \tag{4.87}$$

For k=0,

$$\begin{aligned}\beta_{0,0} &= \int_{-\infty}^{\infty} f_0(x)\psi(x-0)dx \\ &= \int_{-\infty}^{\infty} f_0(x)\psi(x)dx\end{aligned} \tag{4.88}$$

But,



So dilation function $\psi(x)$ varies from 0 to 1.

$= \int_0^{\frac{1}{2}} (x^2)(1) + \int_{\frac{1}{2}}^1 (x^2)(-1)dx$

$= \left[\frac{x^3}{3}\right]_0^{\frac{1}{2}} - \left[\frac{x^3}{3}\right]_{\frac{1}{2}}^1$

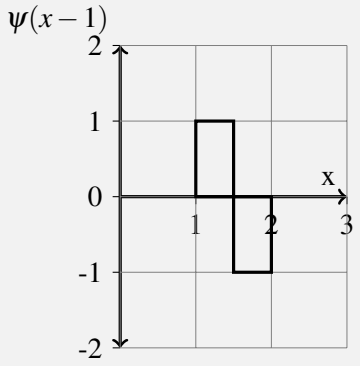
$= -\frac{1}{4}$

$$\boxed{\therefore \beta_{0,0} = -\frac{1}{4}}$$

For k=1,

$$\begin{aligned}\beta_{0,1} &= \int_{-\infty}^{\infty} f_0(x)\psi(x-1)dx \\ &= \int_{-\infty}^{\infty} f_0(x)\psi(x-1)dx\end{aligned} \tag{4.89}$$

But,



So dilation function $\psi(x-1)$ varies from 1 to 2.

$= \int_1^{\frac{3}{2}} (x^2)(1) + \int_{\frac{3}{2}}^2 (x^2)(-1)dx$

$= \left[\frac{x^3}{3}\right]_1^{\frac{3}{2}} - \left[\frac{x^3}{3}\right]_{\frac{3}{2}}^2$

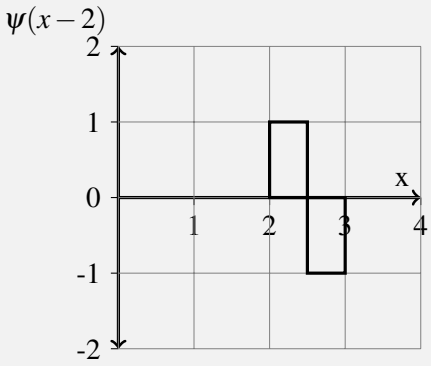
$= -\frac{3}{4}$

$$\therefore \beta_{0,1} = -\frac{3}{4}$$

For k=2,

$\beta_{0,2} = \int_{-\infty}^{\infty} f_0(x)\psi(x-2)dx$

But,



So dilation function $\psi(x-2)$ varies from 2 to 3.

$= \int_2^{\frac{5}{2}} (x^2)(1) + \int_{\frac{5}{2}}^3 (x^2)(-1)dx$

$= \left[\frac{x^3}{3}\right]_2^{\frac{5}{2}} + \left[\frac{x^3}{3}\right]_{\frac{5}{2}}^3$

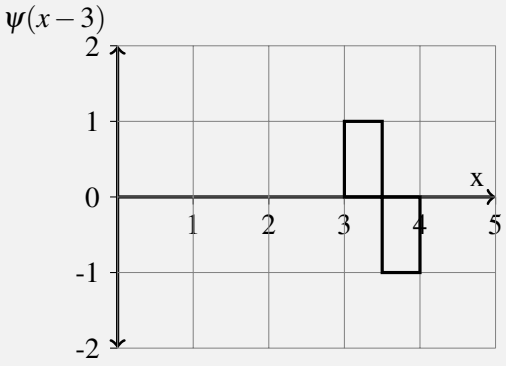
$= -\frac{5}{4}$

$$\boxed{\therefore \beta_{0,2} = -\frac{5}{4}}$$

For k=3,

$\beta_{0,3} = \int_{-\infty}^{\infty} f_0(x)\psi(x-3)dx$

But,



So dilation function $\psi(x-3)$ varies from 3 to 4.

$= \int_3^{\frac{7}{2}} (x^2)(1) + \int_{\frac{7}{2}}^4 (x^2)(-1)dx$

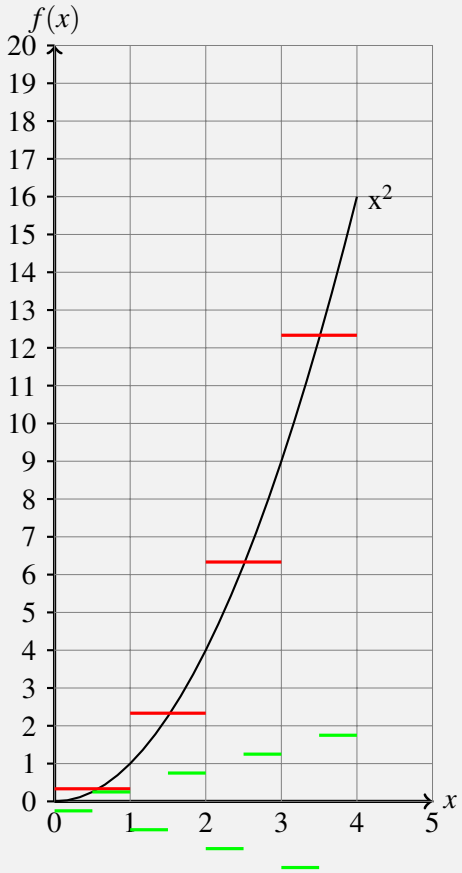
$= \left[\frac{x^3}{3}\right]_3^{\frac{7}{2}} - \left[\frac{x^3}{3}\right]_{\frac{7}{2}}^4$

$= -\frac{7}{4}$

$$\boxed{\therefore \beta_{0,3} = -\frac{7}{4}}$$

Using equation (6),we can write

$g_0(x) = -\frac{1}{4}\psi(x) - \frac{3}{4}\psi(x-1) - \frac{5}{4}\psi(x-2) - \frac{7}{4}\psi(x-3)$



The Red line is showing Wavelet scaling coefficients.
The Green line is showing Wavelet dilation coefficients.

3] $f_1(x) \in v_1 \therefore j = 1$ so $w_a = \frac{1}{2^1} = \frac{1}{2}$

$span\left\{2^{\frac{1}{2}}\phi(2^0x-k)\right\}$

$span\left\{\sqrt{2}\phi(2x-k)\right\}$

From (2),when we put j=1

$f_1(x) = \sum_k \alpha_{1,k}(2)^{\frac{1}{2}}\phi(2^1x-k)$

$= \sum_k \alpha_{1,k}\sqrt{2}\phi(2x-k)$

$\alpha_{1,k} = \int_{-\infty}^{\infty} f_1(x)2^{\frac{0}{2}}\phi(2^0x-k)$

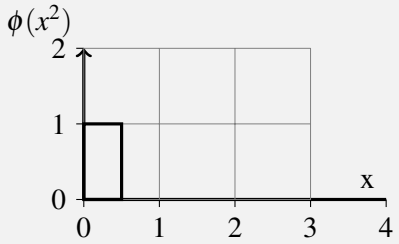
$$\alpha_{1,k} = \int_{-\infty}^{\infty} f_1(x)\phi(2x-k) \tag{4.90}$$

For k=0,

$\alpha_{1,0} = \int_{-\infty}^{\infty} f_1(x)\sqrt{2}\phi(2x-0)dx$

$= \int_{-\infty}^{\infty} f_1(x)\sqrt{2}\phi(x^2)dx$

But,



As scaling function $\phi(x^2)$ varies from 0 to 0.5.

$= \int_0^{\frac{1}{2}} \sqrt{2}(x^2)(1)dx$

$= \sqrt{2}\left[\frac{x^3}{3}\right]_0^{\frac{1}{2}}$

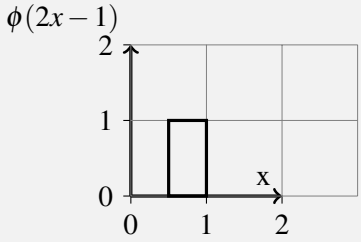
$= \sqrt{2}\cdot\frac{1}{24}$

$$\therefore \alpha_{1,0} = \frac{1}{12\sqrt{2}}$$

For k=1,

$\alpha_{1,1} = \int_{-\infty}^{\infty} \sqrt{2} f_1(x)\phi(2x-1)dx$

But,



As scaling function $\phi(2x-1)$ varies from 0.5 to 1.

$= \int_0^{\frac{1}{2}} \sqrt{2}(x^2)(1)dx$

$= \sqrt{2}\left[\frac{x^3}{3}\right]_0^{\frac{1}{2}}$

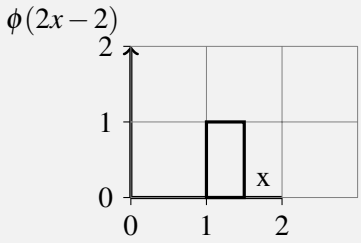
$= \sqrt{2}\cdot\frac{7}{24}$

$$\therefore \alpha_{1,1} = \frac{7}{12\sqrt{2}}$$

For k=2,

$\alpha_{1,2} = \int_{-\infty}^{\infty} f_1(x)\sqrt{2}\phi(2x-2)dx$

But,

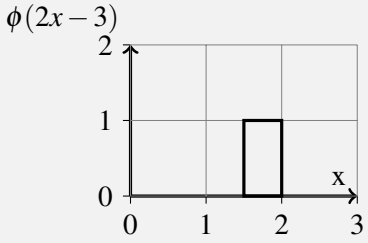


As scaling function $\phi(2x-2)$ varies from 1 to $\frac{3}{2}$.
$= \int_1^{\frac{3}{2}} \sqrt{2}(x^2)(1)dx$
$= \sqrt{2}\left[\frac{x^3}{3}\right]_1^{\frac{3}{2}}$
$= \sqrt{2} \cdot \frac{19}{24}$
$\boxed{\therefore \alpha_{1,2} = \frac{19}{12\sqrt{2}}}$

For k=3,

$\alpha_{1,3} = \int_{-\infty}^{\infty} f_1(x)\sqrt{2}\phi(2x-3)dx$

But,

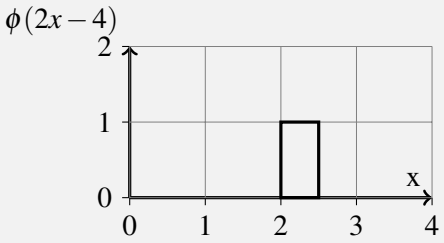


As scaling function $\phi(2x-3)$ varies from 1.5 to 2.
$= \int_{\frac{3}{2}}^{2} \sqrt{2}(x^2)(1)dx$
$= \sqrt{2}\left[\frac{x^3}{3}\right]_{\frac{3}{2}}^{2}$
$= \sqrt{2} \cdot \frac{37}{24}$
$\boxed{\therefore \alpha_{1,3} = \frac{37}{12\sqrt{2}}}$

For k=4,

$\alpha_{1,4} = \int_{-\infty}^{\infty} f_1(x)\sqrt{2}\phi(2x-4)dx$

But,



As scaling function $\phi(2x-4)$ varies from 2 to $\frac{5}{2}$.

$= \int_2^{\frac{5}{2}} \sqrt{2}(x^2)(1)dx$

$= \sqrt{2}\left[\frac{x^3}{3}\right]_2^{\frac{5}{2}}$

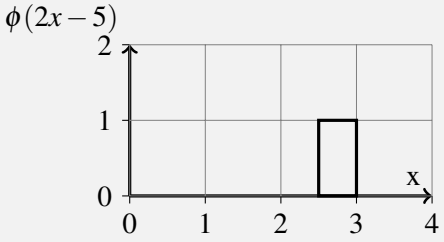
$= \sqrt{2} \cdot \frac{61}{24}$

$$\therefore \alpha_{1,4} = \frac{61}{12\sqrt{2}}$$

For k=5,

$\alpha_{1,5} = \int_{-\infty}^{\infty} f_1(x)\sqrt{2}\phi(2x-5)dx$

But,



As scaling function $\phi(2x-5)$ varies from $\frac{5}{2}$ to 3.

$= \int_{\frac{5}{2}}^{3} \sqrt{2}(x^2)(1)dx$

$= \sqrt{2}\left[\frac{x^3}{3}\right]_{\frac{5}{2}}^{3}$

$= \sqrt{2} \cdot \frac{91}{24}$

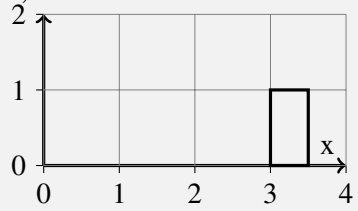
$$\therefore \alpha_{1,5} = \frac{91}{12\sqrt{2}}$$

For k=6,

$\alpha_{1,6} = \int_{-\infty}^{\infty} f_1(x)\sqrt{2}\phi(2x-6)dx$

But,

$\phi(2x-6)$



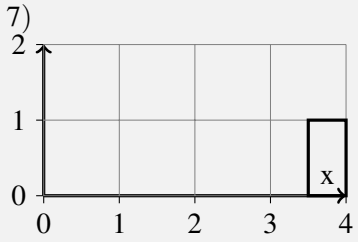
As scaling function $\phi(2x-6)$ varies from 3 to $\frac{7}{2}$.
$= \int_3^{\frac{7}{2}} \sqrt{2}(x^2)(1)dx$
$= \sqrt{2}\left[\frac{x^2}{2}\right]_3^{\frac{7}{2}}$
$= \sqrt{2}\cdot\frac{127}{24}$
$\boxed{\therefore \alpha_{1,4} = \frac{127}{12\sqrt{2}}}$

For k=7,

$\alpha_{1,7} = \int_{-\infty}^{\infty} f_1(x)\sqrt{2}\phi(2x-7)dx$

But,

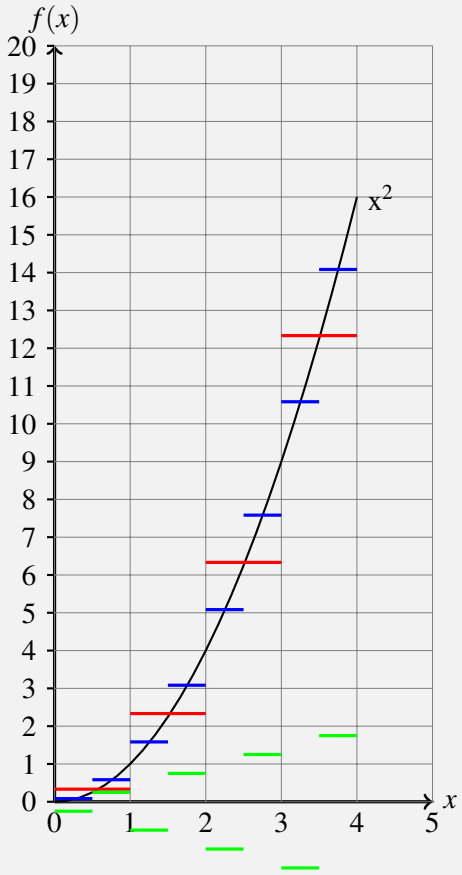
$\phi(2x-7)$



As scaling function $\phi(2x-7)$ varies from $\frac{7}{2}$ to 4.
$= \int_{\frac{7}{2}}^{4} \sqrt{2}(x^2)(1)dx$
$= \sqrt{2}\left[\frac{x^2}{2}\right]_{\frac{7}{2}}^{4}$
$= \sqrt{2}\cdot\frac{169}{24}$
$\boxed{\therefore \alpha_{1,7} = \frac{169}{12\sqrt{2}}}$

$f_1(x) = \frac{1}{12}\phi(2x) + \frac{7}{12}\phi(2x-1) + \frac{19}{12}\phi(2x-2) + \frac{37}{12}\phi(2x-3)$

$+\frac{61}{12}\phi(2x-4)+\frac{91}{12}\phi(2x-5)+\frac{127}{12}\phi(2x-6)+\frac{169}{12}\phi(2x-7)$



The Red line is showing Wavelet scaling coefficients.
The Green line is showing Wavelet dilation coefficients.
The Blue line is showing Wavelet scaling coefficients of different Phi.

4] Prove: $v_1 = v_0 \oplus w_0$
$v_0 = \{\frac{1}{3}, \frac{7}{3}, \frac{19}{3}, \frac{37}{3}\}$

$w_0 = \{\frac{-1}{4}, \frac{-3}{4}, \frac{-5}{4}, \frac{-7}{4}, \}$

$v_1 = \{\frac{1}{12}, \frac{7}{12}, \frac{19}{12}, \frac{37}{12}, \frac{61}{12}, \frac{91}{12}, \frac{127}{12}, \frac{169}{12}\}$

Now, Take first component of v_0 and first component of w_0.
and Find out following term,
$\alpha_{0,1} \cdot \phi(x) + \beta_{0,1} \cdot \psi(x)$
The Scaling function varies from 0 to 1 and the value is 1 over all interval. The dilation function varies from 0 to 1 and the value is 1 for 0 to 0.5 and -1 for 0.5 to 1. So, We will get two values

using these two functions.
For k=0
$\therefore {\alpha_{0,0}}^{(+)} + {\beta_{0,0}}^{(+)}$
$=\frac{1}{3} + \frac{-1}{4}$
$=\frac{1}{12}$
This is equal to $\alpha_{1,0}$.

Also,
$\therefore {\alpha_{0,0}}^{(-)} + {\beta_{0,0}}^{(-)}$
$=\frac{1}{3} + \frac{1}{4}$
$=\frac{7}{12}$

This is equal to $\alpha_{1,1}$.

For K=1
$\therefore {\alpha_{0,1}}^{(+)} + {\beta_{0,1}}^{(+)}$
$=\frac{7}{3} + \frac{-3}{4}$
$=\frac{19}{2}$

This is equal to $\alpha_{1,2}$.

$\therefore {\alpha_{0,1}}^{(-)} + {\beta_{0,1}}^{(-)}$
$=\frac{7}{3} + \frac{3}{4}$
$=\frac{37}{12}$

This is equal to $\alpha_{1,3}$.

Similarly, we can find values of $\alpha_{0,k}$ and $\beta_{0,k}$ to make a conclusion that addition of these two values will $\alpha_{1,k}$.(k=Total number of coefficients).
Thus, v_1 is formed by combining v_0 and w_0.

Hence,$v_1 = v_0 \oplus w_0$.

4.14 Experimental Results - Blind Prakrti ψ

Consider the following function,

$$g(t) = \begin{cases} t & 0 \leq t \leq \frac{1}{2}, \\ t-1 & \frac{1}{2} \leq t < 1 \end{cases} \tag{4.91}$$

We can see that at t=0.5 there is a clear discontinuity. However such discontinuities are easier to detect, we will slightly complicate the matter. We will smooth out this signal by integrating it.

$$h(t) = \int g(t) = \begin{cases} \frac{t^2}{2} & 0 \le t \le \frac{1}{2}, \\ \frac{t^2}{2} - t + \frac{1}{2} & \frac{1}{2} \le t < 1 \end{cases} \tag{4.92}$$

We can see that there is still a cusp jump at t=0.5. Thus integrating again,

$$f(t) = \int h(t) = \begin{cases} \frac{t^3}{6} & 0 \le t \le \frac{1}{2}, \\ \frac{t^3}{6} - \frac{t^2}{2} + \frac{t}{2} - \frac{1}{8} & \frac{1}{2} \le t < 1 \end{cases} \tag{4.93}$$

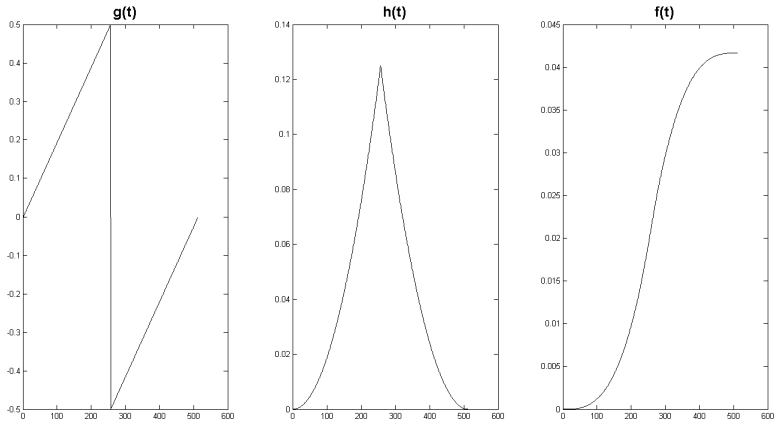


Figure 4.27: Plot of functions g(t), h(t), f(t)

This appears absolutely smooth to eye, but in reality the discontinuity exists.

Simulation Results

As can be seen from figure (4.28), Haar filter completely missed the discontinuity. This is because Haar has only one vanishing moment and can sense only up to first order derivative.

As can be seen from figure (4.29), higher members in Daubechies family can sense and detect the discontinuity. This is because filters like db-2 onwards have at least two vanishing moments and thus can sense up to second order derivative.

This result clearly depicts the blind nature of Prakṛti (प्रकृति) or the ψ function if not chosen correctly using concept of vanishing moments. In compliance with Sāṅkhya (सांख्य) school, this knowledge can be obtained from the seeker, that is ϕ.

4.15 Experimental Results - Age of Vedas

An experimentation is carried out to bring out how vedic age can be approximated using Sāṅkhya (सांख्य) school of thought.

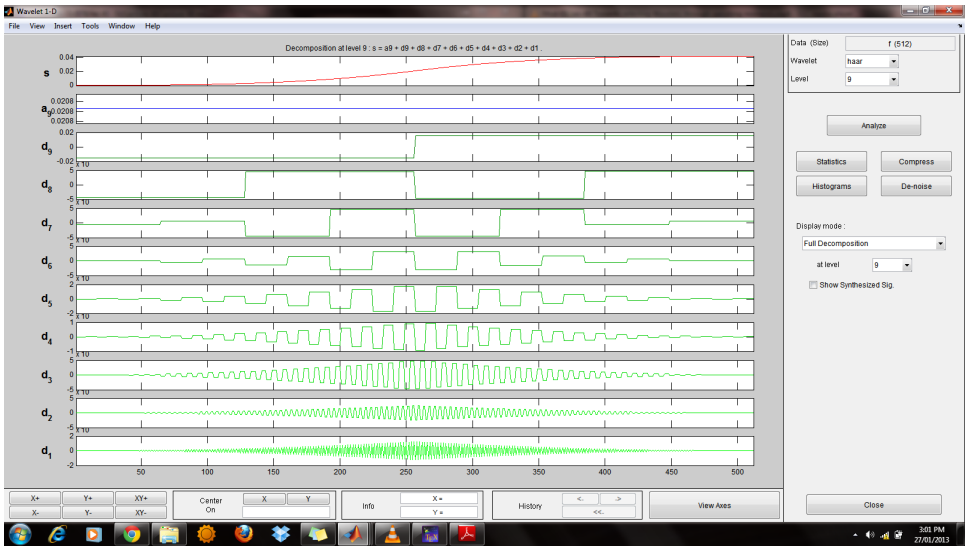


Figure 4.28: Plot of functions g(t), h(t), f(t)

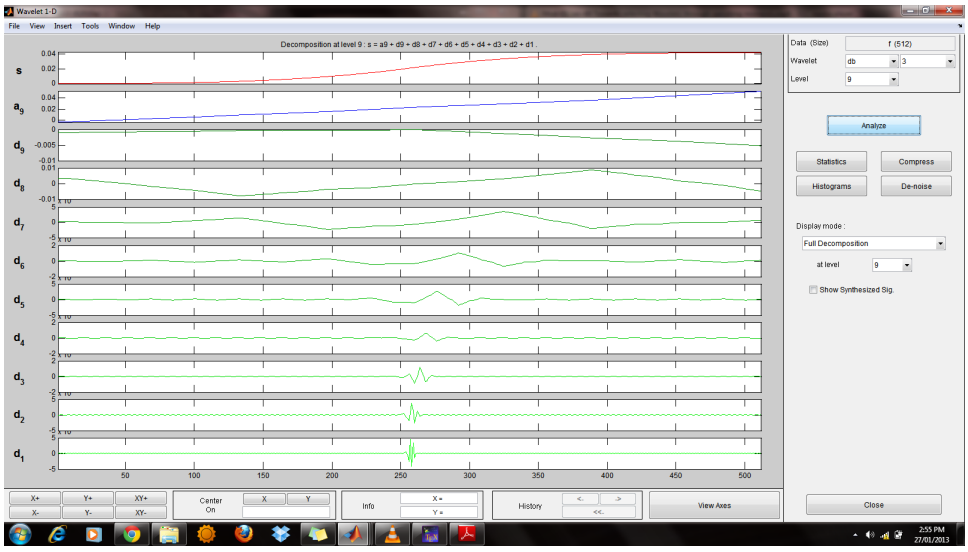


Figure 4.29: Plot of functions g(t), h(t), f(t)

As Sāṅkhya (सांख्य) school of thought has been decoded as scalable wavelet structures in this study it leads to interesting paradigms. Firstly, as the complete wavelet basis are scalable and self-similar, they produce dyadic structures and can be mapped using $3D$ walks. Thus a 2^j scale combined with third dimension makes it a volumetric proposition. The basic volume unit will be $1^3 = 1$ and the next advancement will be $2^3 = 8$. The gradient, i.e. first order derivative gives $8-1 = 7$ as volumetric increments. Thus, Prakṛti (प्रकृति) can be thought of bound by 7 divisions in each direction with scope to get projected in all the 4 quadrants. Thus, $7 \times 4 = 28$ is the total number of sub-division components needed to span the total linear space. This is in line with total of 28 Nakṣatras (नक्षत्रे). In today's almanacs 'Abhijīta (अभिजीत)' is lost and we are left with 27 Nakṣatras (नक्षत्रे). The proof for Vedika (वेदिक) age from Sāṅkhya (सांख्य) school will be as follows:

- The earth spins on its own axis by $1/365.25$ of its annual orbit around the sun.
- The daily spin time is $(24 \times 3600)/365.25 = 236.55$ seconds less than 24 hours.
- For the day time adjustments we need to have standing wave relationship.
- This is exactly as given in Guṇa Sūtra (गुण सूत्र) of Sāṅkhya (सांख्य), which depicts that axiomatically, the rate of variation in cyclic time between any two identical interacting subspaces has to be $1:2$ as shown in Figure(4.30).

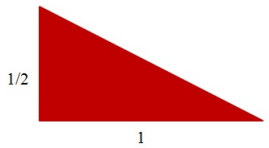


Figure 4.30: The rate of variation in cyclic time between any two identical interacting subspaces has to be $1:2$.

- This ratio can be solved using arctan philosophy, as tangent of an angle of 26.565 is equal to $1/2$.
- At 26.565 degrees the resonant rate of oscillations of standing wave is 296575998. This is identical to velocity of light at meter wavelength.
- Earth is tilted at 23.5 degrees and has mean surface velocity of around 462 m/s and moves with average speed of 29840 m/s around the sun.
- Earth's maximum spin vector is $462 \times 0.91 \times \cos(23.5) = 423.52$
- From this it can be found that 25740 cyclic years will be needed to repeat particular position of earth's orbital.
- 25740 divided by 28 Nakṣatras (नक्षत्रे) yield around 920 years of Nakṣatras (नक्षत्र) cycle.
- Nakshtras Nakṣatras (नक्षत्र) traversed between Punarvasū (पुनर्वसू) (taken as reference) and Aśvinī (अश्विनी) being 7, time elapsed will be $7 \times 920 = 6440$ years.
- total sectors will be $28 + 7 = 35$, thus giving $35 \times 920 = 32200$ years.
- We are in 2014, reducing that $32200 - 2014 = 29986$.
- This comes from the fact that it will take about 32200 years of full cycle to trace back recurring position of Punarvasū (पुनर्वसू).
- This proves Vedic age to be around 30000 BC and it is in compliance with the arguments put forth by Lokamānya Ṭiḷaka (लोकमान्य टिळक) in his book 'The Arctic Home in the Vedas' [[54]].

4.16 Summary

- This chapter presents a sound mathematical framework to bring out the wavelet conjunctures from the Sāṅkhya (सांख्य) as well as Vedānta (वेदान्त) school of thought.
- This work clearly explains Paṅgu-Aṃdha Nyāya (पंगु - अंध न्याय) of Sāṅkhya (सांख्य) Philosophy with concrete examples from modern theory.
- This work, explains how the Sarga (सर्ग) proclaimed in the Sāṅkhya (सांख्य) philosophy fits in with the concept of Multi-resolution Analysis of Wavelet domain.
- This work uses the scalable and self-similar wavelet composition extracted from Sāṅkhya (सांख्य) school to approximate the Vedic age with good accuracy.
- From Sāṅkhya (सांख्य) we could elevate our understanding to Vedānta (वेदान्त).
- MRA (Multi-Resolution-Analysis) framework subtly deploys Adhyāsavāda (अध्यासवाद) or Vivartavāda (विवर्तवाद).
- The building blocks of the MRA framework were demonstrated to work well through conceptualization of the notions of Śapala Brahma (शपल ब्रह्म) as $\phi(.)$ and Māyā (माया) as $\psi(.)$.
- The theme is also proved using अनिर्वचनीय ख्याति.

5. Concluding Remarks

5.1 Contributions of Thesis

- This thesis brings out a novel and unique interpretation of Nāsadīya Sūkta using energy analysis by building signal processing perspective.
- This work connects the 'Origin of Universe' as prescribed in Nāsadīya Sūkta with the modern science and the allied theories.
- The energy analysis is validated by applying the same to few other Sūktas apart from the Nāsadīya Sūkta.
- The analysis is not only put forth mathematically and analytically, but it is also validated through experimentation.
- The experimentation is carried through analytical building blocks as well as simulations.
- The experimentation is carried out on Sāṅkhya and Vedānta systems.
- While performing the experimentation the filter banks based systems are built and tested.
- Few intriguing concepts like 'Paṅgū-Andha-Nyāya' (पङ्गू - अन्ध - न्याय) from Sāṅkhya is established and validated through simulations.
- The results of simulations are presented as specific case study.
- Several such simulations are presented and the codes are made available for the readers to try the things out themselves.
- Every hymn in Nāsadīya Sūkta is decoded systematically.
- The meaning of every word is given and then a combined comprehension is brought out that connects the dots to complete the big picture.
- Special discussion section after every hymn to bring out the correspondence with the modern science is another unique feature of this work.
- This work uses the scalable and self-similar wavelet composition extracted from Sāṅkhya (सांख्य) school to approximate the Vedic age with good accuracy.
- From Sāṅkhya (सांख्य) we could elevate our understanding to Vedānta (वेदान्त).
- MRA (Multi-Resolution-Analysis) framework subtly deploys Adhyāsavāda (अध्यासवाद) or Vivartavāda (विवर्तवाद).

- The building blocks of the MRA framework were demonstrated to work well through conceptualization of the notions of Śapala Brahma (शपल ब्रह्म) as $\phi(.)$ and Māyā (माया) as $\psi(.)$.
- The theme is also proved using अनिर्वचनीय ख्याति.
- This work presents the mathematical proofs of the doctrines proposed and this feature of the work makes the drawings more robust.
- Novel meaning of Big-Bang is brought out with reference to Verse-5 of Nāsadīya Sūkta.
- This work also brings out the exact difference between Dvandva (द्वन्द्व) and Dvaita (द्वैत) and encodes the same mathematically.
- This work also hence proves the importance of Advaita (अद्वैत) in the root cause of the creation of the universe.

5.2 Conclusions form the experimentation

- Nāsadīya Sūkta presents as very systematic but very subtle schema of understanding 'Origin of the Universe'.
- Nāsadīya Sūkta strongly relates with the modern science particularly with Quantum Physics and The-Big-Bang-Theory in general.
- The Vaidika literature seams to have captures the theory of relativity long back in a very prudent way.
- The energy analysis works very well for deciphering the cryptic verses and presents the crisp decoded samples.
- The energy analysis as prescribed by Yāska (यास्क) solves many mysteries beneath like 'Paṅgū-Andha-Nyāya' (पङ्गू - अन्ध - न्याय).
- The joint-time-frequency perspective as captured by the Vaidika literature and as per the modern Wavelet theory resemble closely.
- The doctrine of 'Satkāryavāda' (सत्कार्यवाद) as prescribed by Sāṅkhya as well as Vedānta branches of philosophy are proved.
- This is an important finding as the hymn begins with 'Na-Sat...' (नासद् = न सत्) and hence may appear to be against 'Satkāryavāda' (सत्कार्यवाद).
- The verses of Nāsadīya Sūkta also manifest beautifully in later works including Upaniṣadas (उपनिषदे), Bhagavadgītā (भगवद्गीता) and Jñāneśvarī (ज्ञानेश्वरी).
- The deep study of Vaidika literature presents the comprehensive picture of the Universe which also harmonizes with the modern science. The advanced findings may also give direction to the modern science.

5.3 Future Directions

Any good research gives rise to many interesting questions. This research has given many future directions, few of which are as follows:

- It is desired to explore this subject from perspective apart from Energy analysis.
- Other Sūktas from energy analysis can also be good topic of study.
- There are certain mysteries which are left as future work: e.g. divinity in number 432. Ṛgveda has 432,000 syllables. Square of 432 gives the approximated speed of light! The diameter of sun = 864,000 miles and $864 = 432 \times 2!$ $432 \div 4 = 108$ is a special number too! One Mahāyuga is 432000 solar years!
- Furtherance as per three worlds depicted by Yāskācārya is desired.

- This energy analysis can be taken to another spaces like Hilbert spaces to prove other doctrines in modern physics.

5.3.1 In search of scaling and wavelets coefficients

We have already shown that $\psi(.)$ or Māyā (माया) function emerges from the $\phi(.)$ or Śapala Brahma (शपल ब्रह्म) function. Therefore, once the trick to figure of $\phi(.)$ is understood, it unlocks the entire ladder that follow. We have discussed so far Haar and Daubechies wavelets families. The coefficients of low pass and high pass filters for these are well known. However where exactly we get these coefficients from? Do we have a concrete procedure for finding them? What properties should the scaling equation obey? These are few questions we would try to answer now.

Scaling function is also 'Brahma' (ब्रह्म) function and the entire universe is a manifestation of the same. Finding the kernels which satisfy the norms and become 'Brahma' (ब्रह्म) functions is a interesting quest. This sub-section provides the directions for finding the kernel function. This certainly has huge potential for further research.

We are familiar with Haar wavelet and scaling function is given as,

$$\psi(t) = \begin{cases} 1 & 0 \leq x < \frac{1}{2} \\ -1 & \frac{1}{2} \leq x < 1 \\ 0 & \text{otherwise} \end{cases}$$

$$\psi(t) = \begin{cases} 1 & 0 \leq x < 1 \\ 0 & \text{otherwise} \end{cases}$$

However important question is where did the above values come from? We will now focus on answering this particular question.
The dilation equation in time domain is given as,

$$\phi(t) = \sum_k h_k \sqrt{2} \phi(2t-k) \tag{5.1}$$

$$\psi(t) = \sum_k g_k \sqrt{2} \phi(2t-k) \tag{5.2}$$

From equations (4) and (5) we can see that $\phi(t) \in V_0$, $\psi(t) \in W_0$, $\phi(2t-k) \in V_1$ and $\psi(2t-k) \in W_1$. Also we know that, $V_1 = V_0 \oplus W_0$. Thus once we know $\phi(2t-k)$ we can find out $\psi(t)$ and $\phi(t)$, hence focus is on finding out scaling equation coefficients.

Now in order to achieve this lets go through three guiding theorems.

Theorem I:
For the scaling equation $\phi(t) = \sum_k h_k \sqrt{2} \phi(2t-k)$, with non-vanishing coefficients $\{h_k\}_{k=N}^{M}$ only for $N \leq k \leq M$ its $\phi(x)$ is with a compact support contained in the interval [N,M].

Theorem II:

If the scaling function $\phi(x)$ has compact support on $0 \leq x \leq N-1$ and if, $\{\phi(x-k)\}$ are linearly independent, then $h_n = h(n) = 0$, for $n < 0$ and $n > N-1$. Hence N is the length of the sequence.

Theorem III:
If the scaling coefficients $\{h_k\}$ satisfy the condition for existence and orthogonality of $\phi(t)$, then
$\psi(x) = \sum_k \sqrt{2}\phi(2x-t)$
where, $g_k = \pm(-1)^k h_{N-k}$
and $\int_{-\infty}^{\infty} \psi(x-l)\phi(x-k)dx = \delta_{1,k} = 0, l \neq k$

From these theorems, we will try to find out coefficients of scaling equation. In order to be able to do that following are some properties that scaling coefficients have to obey,

1. $\sum h_k = \sqrt{2}$

2. $\sum h_{2k} = \frac{1}{\sqrt{2}}$

3. $\sum h_{2k+1} = \frac{1}{\sqrt{2}}$

4. $\sum |h_k|^2 = 1$

5. $\sum h_{k-2l} h_k = \delta_{l,0}$

6. $\sum 2h_{k-2l} h_{k-2j} = \delta_{l,j}$

Now what is challenging is to understand from where exactly we get these properties from. We would first delve on that and then we would solve one example with case study of Haar $\phi(t)$.

Consider property 1 of $\phi(t)$,

$$\sum_k h_k = \sqrt{2}$$

This property is dependent on what kind of normalization we make use of and rest of the properties are in a way dependent on this property. So we would first understand from where this property comes from.
We know the dilation equation,

$$\phi(t) = \sum_k h_k \sqrt{2}\phi(2t-k)$$

Integrating both the sides,

$$\int_{-\infty}^{\infty} \phi(t)dt = \sum_k h_k \sqrt{2} \int_{-\infty}^{\infty} \phi(2t-k)dt$$

$$\text{Put, } 2t-k = x \Rightarrow dt = \frac{dx}{2}$$

$$\int_{-\infty}^{\infty} \phi(t)dt = \sum_k h_k \sqrt{2}\frac{1}{2} \int_{-\infty}^{\infty} \phi(t)dt$$

Let, $\int_{-\infty}^{\infty} \phi(t)dt$ which is normalization be 1. Then we are left with,

$$1 = \sum_k h_k \sqrt{2} \frac{1}{2}$$

$$\sum_k h_k = \sqrt{2} \tag{5.3}$$

The point to note is that equation (6) holds true if we consider the normalization to be 1.

At this point let us assume that property 5 hold true and we would use it to derive the remaining properties. The property number 5 tells us that,

$$\sum_k h_{k-2l} h_k = \delta_{lo}$$

Replacing l by -l we have,

$$\delta_{-lo} = \sum_{k=-\infty}^{\infty} h_{k+2l} h_k$$

Separating 'even' and 'odd' terms,

$$\delta_{-lo} = \sum_{k=-\infty}^{\infty} h_{2k+2l} h_{2k} + \sum_{k=-\infty}^{\infty} h_{2k+1+2l} h_{2k+1}$$

$$\sum_{l=-\infty}^{\infty} \delta_{-lo} = \sum_{l=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} [h_{2k+2l} h_{2k} + h_{2k+1+2l} h_{2k+1}]$$

However we know that,

$$\sum_{l=-\infty}^{\infty} \delta_{-lo} = 1$$

Thus,

$$1 = \sum_{k=-\infty}^{\infty} h_{2k} [\sum_{l=-\infty}^{\infty} h_{2k+2l}] + \sum_{k=-\infty}^{\infty} h_{2k+1} [\sum_{l=-\infty}^{\infty} h_{2k+1+2l}] \tag{5.4}$$

By substituting $l = l - k$ we can write,

$$\sum_{l=-\infty}^{\infty} h_{2k+2l} = \sum_{l=-\infty}^{\infty} h_{2l} = A \tag{5.5}$$

$$\sum_{l=-\infty}^{\infty} h_{2k+1+2l} = \sum_{l=-\infty}^{\infty} h_{2l+1} = B \tag{5.6}$$

From equations (7), (8) and (9) we can write,

$$1 = AA + BB \tag{5.7}$$

$$1 = A^2 + B^2 \tag{5.8}$$

Which clearly represents equation of a circle with radius 1. However there are two unknowns A and B and we have just equation. So we need another equation. This equation comes out of the first property i.e.

$$\sum_k h_k = \sqrt{2}$$

Splitting 'even' and 'odd' terms,

$$\sqrt{2} = \sum_k h_{2k} + \sum_k h_{2k+1}$$

$$\sqrt{2} = A + B \tag{5.9}$$

At this point we have an interesting situation equation (11) represents a circle and (12) a line. The situation is as shown in figure.

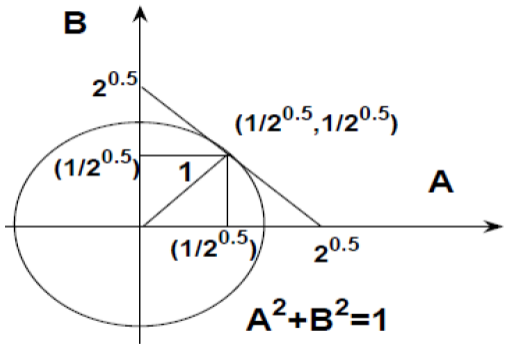


Figure 5.1: Solution to equation (11) and (12)

Hence from figure we have,

$$A = \frac{1}{\sqrt{2}}$$

$$B = \frac{1}{\sqrt{2}}$$

Thus by substituting A we have,

$$\sum_k h_{2k} = \frac{1}{\sqrt{2}} \tag{5.10}$$

which gives us property number 2. Also substituting the value of B we have,

$$\sum_k h_{2k+1} = \frac{1}{\sqrt{2}} \tag{5.11}$$

which gives property number 3.

Now consider property number 4 from equation (11) we have,

$$\sum_k |h_k|^2 = \sum_k h_k^2 = 1 \tag{5.12}$$

Now lets verify these properties in case of Haar scaling function. We know that in case of Haar $\phi(t)$,

$$h_0 = h_1 = \frac{1}{\sqrt{2}}$$

Putting these values in property 1 we have,

$$\sum_0^1 h_k = h_0 + h_1 = \sqrt{2}$$

Thus property 1 is obeyed.
Similarly for property 2, 3 and 4,

$$\sum_k h_{2k} = h_0 = \frac{1}{\sqrt{2}}$$
$$\sum_k h_{2k+1} = h_1 = \frac{1}{\sqrt{2}}$$
$$\{\sum_k h_{2k}\}^2 + \{\sum_k h_{2k+1}\}^2 = h_0^2 + h_1^2 = 1$$

Hence we can clearly see that these properties are obeyed. For property number 5 we have,

$$\sum_k h_{k-2l} h_k = \delta_{0,l}$$

We have only two indices i.e. 0,1. Lets take the first case,

Case (i)
$k-2l=0$ or $k=2l$
if $l \neq 0$, then $k=2l \geq 2$. Thus sum vanishes unless $l=0$ and in that case $k=0$.
Thus we can write,

$$\sum_k h_k - 0h_k = h_0 h_0 = \frac{1}{2} \tag{5.13}$$

Also when $k=0$, $h_{k-2l} = h_{0-2l}$, which is 0 unless $l=0$. Thus,

$$\sum_k h_k h_k = h_0^2 = \frac{1}{2} \tag{5.14}$$

From equations (16) and (17) we can see that the property 5 is obeyed since their results add up to give 1.

Case (ii)
$k-2l=1$, $k=2l+1$
if $l \neq 0$, $k \geq 3$. Thus l has to be 0 and $k=1$. Hence we have,

$$\sum_k h_k h_k = h_1^2 = \frac{1}{2} \tag{5.15}$$

Now for $k=1$, $l=0$ hence,

$$\sum_k h_k - 0h_k = h_1 h_1 = \frac{1}{2} \tag{5.16}$$

Similar to case (i), equation (18) and (19) add up to give 1 hence for case (ii) also property 5 is obeyed. Point to note is the property 5 gives us idea about the orthogonality and not all the functions obey this property.
Hence the Haar scaling coefficients obey all these different properties. Also once we find out these scaling coefficients we can find out coefficients of wavelet equation as follows,

$$g_k = (-1)^k h_N - k$$

For N=1 we have,

$$g_k = (-1)^k h_1 - k$$

For $k=0$,

$$g_0 = (-1)^0 h_1 = h_1 = \frac{1}{\sqrt{2}}$$

For $k = 1$,

$$g_1 = (-1)^1 h_0 = -h_0 = \frac{-1}{\sqrt{2}}$$

This is how we can find out coefficients of scaling and wavelet equation.

5.3.2 Wavelets and Self-Similarity

Wavelets or Māyā (माया) functions are often used to determine self-similarity amongst signals. This makes sense as Wavelets themselves are structures depicting self-similar nature and hence are the natural choice for such type of applications. The Vedānta schools of philosophy also projects the entire world to be Pratibimba (प्रतिबिम्ब) of this Māyā (माया) function. Hence, exploring self-similarity in these kernels is a potential area of further research.

Conventionally signal processing makes use of robust LSI (Linear Shift Invariant) structure for analyzing various types of signals. For particular signals, which are self similar, a different approach is required.

When the signals in local segments are similar in specific way to the entire signal, they are called as self-similar signals. The temporal scaling of such signals produces the signal if it is deterministic else produces statistical characteristics in case of stochastic signals. In literature many examples of self-similar structures have been reported with the likes of Fractional Gaussian noise, homogeneous signals, fractional Brownian motion etc.

Dilation provides the foundation to achieve scale changes in Wavelet analysis. In fact, 'Dilation' and 'Dilation Equations' are truly at the heart of the Multi-resolution framework we have seen since chapter 2. It is through the dilation equation we create a linkage to use 2-band or M-band filter bank structure to create the deployment platform for wavelet filters.

For many computer vision problems it is desired to have scale invariance. There could be two different images captured with different zoom or actual physical distance and yet the application may demand correct match and just the difference in the scales should not spoil the show.

Since dilation is inherent in any wavelet transformation, it leads to hidden **self-similarity** and in a way the fractals with self-similar structures lead to scale-invariant systems. One easier way of understanding scale-invariant systems is when an input is scaled by some scale, the output also gets scaled equally. Such scale-invariant systems (SIS) can be represented as:

$$S[x(t/a)] = y(t/a) \tag{5.17}$$

for $a > 0$. The scaling happens with reference to the independent variable, which is time t is this case. Thus, to gain scale-invariance, one has to give up time-invariance.

Let $k(t,\tau)$ be the system kernel which is function of t and τ both and characterizes linear but time varying system. For this kernel function to correspond to linear and time varying system, the necessary and sufficient condition is:

$$k(t,\tau) = a \times k(a \cdot t, a \cdot \tau) \tag{5.18}$$

System function can be written as,

$$y(t) = \int_{\tau} k(t,\tau) \cdot x(\tau) d\tau \tag{5.19}$$

Thus, the output is obtained by taking dot product between input and the kernel. For $\tau \neq 0$ the kernel $k(t,\tau)$ gets scaled at every instance, thus giving us,

$$\tau k(t,\tau) = h(t,\tau) \tag{5.20}$$

Given this,

$$y(t) = \int_{-\inf}^{\inf} x(\tau)h(t/\tau)\frac{d\tau}{\tau} = \int_{-\inf}^{\inf} x(t/\tau)h(\tau)\frac{d\tau}{\tau} \tag{5.21}$$

For a deterministic signal $x(t)$, it is self-similar if

$$x(t) = a^{-H}x(at) \tag{5.22}$$

OR

$$x(t/a) = a^{-H}x(t) \tag{5.23}$$

for $a > 0$

A random process $X(t)$, is self-similar if the mean (zeroth moment) $M_X(t)$ and autocorrelation $R_X(t,s)$

$$M_X(t) \equiv E[X(t)] = a^{-H}M_X(at) \tag{5.24}$$

and

$$R_X(t,s) \equiv E[X(t)X(s)] = a^{-2H}R_X(at,as) \tag{5.25}$$

for $a > 0$

In the MRA framework we have already seen the scaling of the wavelet kernel through dilation equation. Through the wavelet dilation equation the wavelet function gets connected with father equation, which is also called as *scaling* equation.

This leads to many hidden structures in every wavelet and its scaling function which are scaled down versions of the bigger signal and make the complete structure self similar. This is illustrated in figure (5.2) below.

Because of this very unique property, scaling and wavelet functions can be used to detect self-similarity in signals. Interested readers can explore this further as this has started gaining importance in research community.

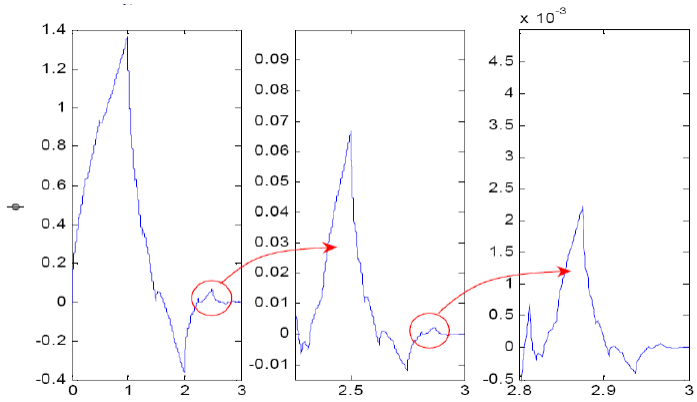
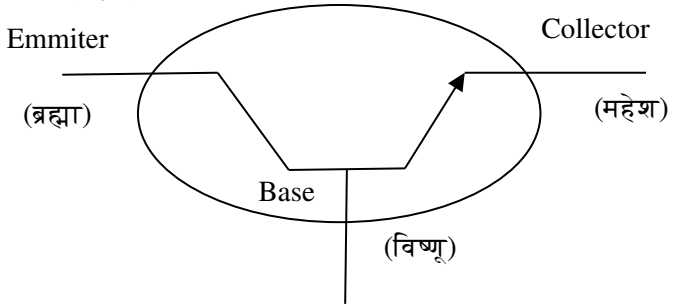


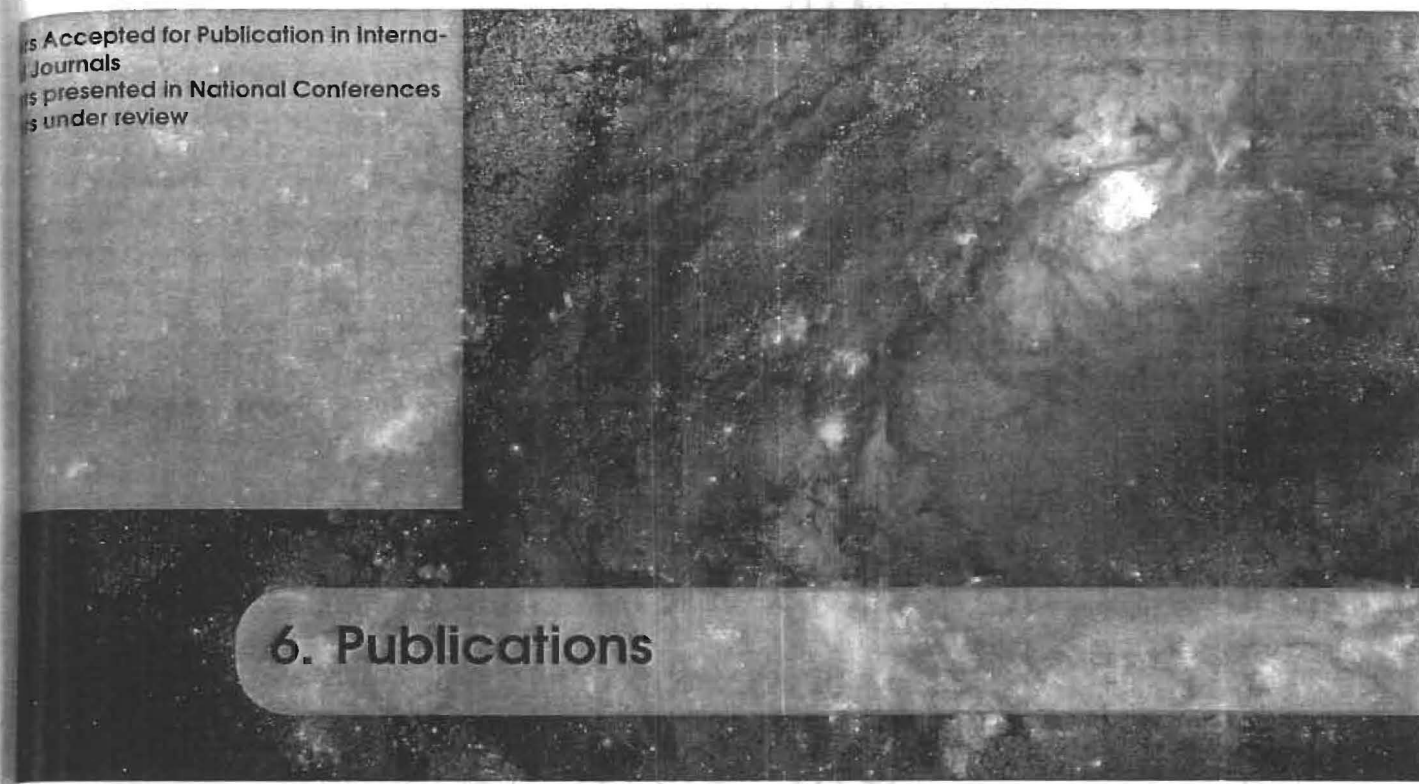
Figure 5.2: Daub-4 scaling function with hidden self-similarity.

5.3.3 Transistors and three deities

It will be a potential area of research to explore remarkable similarity between the three ports of a transistor (in electronic circuitry) and the three deities namely 'Brahmā' (ब्रह्मा), 'Viṣṇū' (विष्णू) and 'Maheśa' (महेश).



The 'emitter' emits the electronics and this is similar to 'Brahmā' (ब्रह्मदेव) starting the operations of the Universe. The 'base' controls the rate of flow of the electrons, which is similar to the functioning of 'Viṣṇū' (विष्णू). The 'collector' essentially collects the electrons, which is similar to the function of 'Maheśa' (महेश) which is responsible for the 'Pralaya' (प्रलय). The similarity is astounding and needs further probing.



6. Publications

6.1 Papers Accepted for Publication in International Journals

- "Towards Comprehending the Third Verse of Nāsadīya Sūkta", Aditya Abhyankar, Shripad Bhat, Accepted for publication in *Wisdom Herald.*
- "Towards Comprehending the Fourth Verse of Nāsadīya Sūkta", Aditya Abhyankar, Shripad Bhat, Accepted for publication in *Universal Review.*
- "Towards Comprehending the Fifth Verse of Nāsadīya Sūkta", Aditya Abhyankar, Shripad Bhat, Accepted for publication in *Parisheelan.*

6.2 Papers presented in National Conferences

- "Evolution of Sāṅkhya (सांख्य) Concepts: A Signal Processing Perspective", Aditya Abhyankar, Shripad Bhat, Presented in *National Seminar on Sanskrit Sources of Indian Hostory (III)* at TMV in 2014.
- "Towards understanding Universe and its Cognition through Vedānta (वेदान्त)", Aditya Abhyankar, Shripad Bhat, Presented Abstract for *National Seminar on Sanskrit Sources of Indian Hostory (IV)* at TMV in 2015.

6.3 Papers under review

- "Towards Comprehending the first Two Verses of Nāsadīya Sūkta", Aditya Abhyankar, Shripad Bhat, submitted to *Indian Journal of Traditional Knowledge.*
- "Towards Comprehending the Sixth Verse of Nāsadīya Sūkta", Aditya Abhyankar, Shripad Bhat, submitted for publication to *Wisdom Herald.*
- "Towards Comprehending the Seventh Verse of Nāsadīya Sūkta", Aditya Abhyankar, Shripad Bhat, submitted for publication to *Universal Review.*

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